A STUDY ON MIMO BEAMFORMING FOR WIRELESS COMMUNICATION SYSTEMS IN FREQUENCY-SELECTIVE FADING CHANNELS

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A STUDY ON MIMO BEAMFORMING FOR WIRELESS COMMUNICATION SYSTEMS IN FREQUENCY-SELECTIVE FADING CHANNELS

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ABSTRACT

In recent years, multiple-input multiple-output (MIMO) antennas systems have quickly become an inevitable wireless technology not only for WLANs but also for mobile networks. Based on the benefits of using multiple transmit antennas and multiple receive antennas, MIMO systems can provide high quality and high speed services. However, for high speed transmission, frequency-selective fading (FSF) is a factor that will degrade the system performance and may cause significant link failure in a broadband wireless communications environment. Two architectures have been investigated for MIMO system to mitigate the effect of FSF. The first architecture is transmission of multiple data streams through spatial multiplexing such as MIMO OFDM (Orthogonal Frequency Division Multiplexing), MIMO SDM (Spatial Division Multiplexing). The second architecture is transmission of a single data stream such as MIMO system based on the transmit diversity technique or using Tapped Delay Line (TDL) structure. Unfortunately, those proposed schemes for both single and multiple data streams in FSF are still a lot of complexities concerning the configuration. In principle, they are not yet to show how many delayed channels should be cancelled by a MIMO system. This dissertation gives insight into the weights determination scheme for a simple MIMO beamforming configuration with assumption of perfect channel state information at both the transmitter and receiver. Based on our proposed iterative update algorithm, the optimal transmit and receive weight vectors are determined to apply to MIMO
frequency-selective fading channels that will maximize the output SINR (Signal to Interference and Noise Ratio) and show maximum number of delayed channels cancellation.

Firstly, the dissertation presents an issue problem of a single data stream transmission using MIMO beamforming scheme. An iterative update algorithm for the transmit and receive weight vectors is determinated in a single user case. Using this method, the degree of freedom (DOF) of both the transmit and the receive antennas are joined together to suppress the delayed channels and maximize the output SINR. The maximum number of delayed channels cancellation is analyzed based on the DOF and confirmed by the computer simulation results.

Next, an iterative update algorithm for transmit and receive weight vectors for multiuser system has been studied to apply to MIMO frequency-selective fading channels. Based on the proposed iterative algorithm, improvement of output SINR and maximum number of delayed channels cancellation are ascertained by our analysis and results of computer simulation.

Finally, based on assumption of the delayed channels are known as well as the desired channel, a spatial-temporal adaptive MIMO beamforming scheme has proposed to improve the system performance by utilizing multiple delayed versions of the transmitted signal. Numerical results demonstrate in comparison with simulation results to validate the analysis.
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Chapter 1

Introduction

1.1 Context of Work

Wireless communications networks are growing from 3G toward a new generation, which provide high quality and high speed services. For the demands in broadband wireless access technologies such as mobile internet, multi-media, however, one of the main problems addressed in wireless communications is signal distortion. It can be classified as Inter-Symbol Interference (ISI) due to the signal delay of going through the multipath fading and Co-Channel Interference (CCI) due to the multiple accesses that will degrade the performance of system and may cause significant link failure in a wireless communications environment [1]-[8].

Adaptive array is one of the most expected ways that enables to improve the performance of wireless communication system. In particular, an important feature is its capability to cancel CCI independent of the angle of arrival. The more antenna elements in an array, the more degrees of freedom (DOF) of array possess to combat the interference and mulitpath fading. For instance, an $N$-element antenna array has $(N - 1)$ DOF and thus can cancel $(N - 1)$ CCIs independent of the multipath environment. For ISI, however, since the conventional adaptive arrays using narrowband beamformer processes the received signal only in spatial domain, it cannot treat the delayed versions of the transmitted signal as separated signals. The solution to the problem is to keep the arrays from having to use its spatial processing in combination with temporal filter using tapped delayed line (TDL) structure [9, 10]. It is also referred to as wideband beamformer adaptive arrays. Therefore, spatial and temporal equalizer based on an antenna array will become
a breakthrough technique, which has capability to suppress effectively both CCI and ISI. Much research on spatial and temporal signal processing using an antenna array at base station (BS) has been proposed. Several adaptive algorithms for deriving the optimal weight vector in the time domain such as Least Mean Squares (LMS), Recursive Least Squares (RLS), and Constant Modulus Algorithm (CMA), which have been illustrated in chapter 2, are viewpoints of extending techniques of spatial and temporal digital equalizer. However, resolving simultaneously both the CCI and the ISI is a difficult task for spatial and temporal adaptive array (STAA) since it requires recursive computation slow convergence in searching for optimum weight vector.

Multiple antenna structure divided into two groups has been of special interest, particularly in the last two decades. They are including: use of antenna array only at receiver, known as single-input multiple-outputs (SIMO) systems; and use of antenna only at transmitter, known as multiple-inputs single-output (MISO) systems. However, in order to satisfy the needs of the high performance and capacity of wireless communications system, use of antenna arrays at both transmitter and receiver, known as multiple-input multiple-output (MIMO) systems have been proposed in recent years. If multipath scattering is sufficiently rich and properly exploited, MIMO systems show high performance and capacity compared with SIMO and MISO systems. While most researches have considered the theoretical capacity and output maximum Signal-to-Noise and Interference Ratio (SINR) of MIMO systems in flat fading environment [26]-[38], the implementation issues in wideband MIMO systems for frequency-selective fading environment are still a challenging topic which needs to be resolved.

For solving the MIMO frequency-selective fading channels, spatial multiplexing OFDM has been proposed to transmit multiple independent streams simultaneously, where the number of independent streams is limited by the minimum number of antenna elements at both ends thus frequency-selective MIMO channels are transformed into several frequency-flat MIMO channels. And besides, the decision feedback equalization technique, where MIMO antennas systems equipped with tapped delayed line (TDL) structure, have been proposed for mitigating the frequency-selective fading. However, they so far still suffer from computational complexity, compact and low-cost hardware. Although there are many techniques
1.2 Original Contributions

proposed for MIMO frequency-selective fading environment with or without prior knowledge of Channel State Information (CSI) at the transmitter and/or receiver, they do not yet point out how many delayed channels can be effectively cancelled by using adaptive beamforming by adjusting both the transmit and receive weight vectors in the MIMO system. Therefore, the solution to the problem is to determine the optimal transmit and receive weight vectors for MIMO frequency-selective fading without using the TDL equipment.

With \( M \) and \( N \) antenna elements equipped at the transmitter and receiver, respectively, the weight vectors determination scheme for the transmitter and receiver of a MIMO beamforming is performed by utilizing spatial filter to mitigate CCI and ISI and maximize the output SINR. On the other hand, our proposed scheme has a capability to reduce computational complexity and achieve faster convergence rate compared with MIMO systems having TDL structure in the transmitter and/or receiver.

The original contributions of our work are presented in the next section.

1.2 Original Contributions

Several contributions on the weight vectors determination for the transmitter and receiver of the MIMO beamforming configuration and its performance has been made in this work. Parts of these contributions have been published or submitted for publication. The following list summarizes our main contributions within the scope of this work.

- Firstly, a detailed weight vectors for transmitter and receiver of a MIMO beamforming scheme in frequency-selective fading environment is presented in Chapter 4. The result of analysis was published in the *IEICE Transaction on Communications*, vol.E87-B, no.8, August, 2004.

- Second is the detailed weight vectors for multiuser system of a MIMO beamforming in frequency-selective fading channels performed in Chapter 5. The result of the analysis was published in the *IEICE Transaction on Fundamentals, Special Section on Adaptive Signal Processing and Its Applications*, vol.E88-A, no.3, March, 2005.
• Third is an extensible configuration of MIMO beamforming that improves the performance of system by utilizing multiple delayed versions of the transmitted signal instead of suppressing them. The result of the analysis is submitted to \textit{IEICE Transaction on Communications} (Under 1\textsuperscript{st} Review)

1.3 Structure of the Thesis

The organization of this thesis is as follows.

We begin by describing fundamentals of adaptive arrays in Chapter 2. The basic concepts and classification of adaptive arrays are presented. Then the array signal model in multipath fading environments is developed. Criteria to optimize performance of adaptive arrays and adaptive algorithms to obtain optimal weight vector are also summarized.

In Chapter 3, such a single data stream and multiple data streams transmissions using a single carrier are presented providing a fundamental understanding of the MIMO beamforming with the perfect CSI at both sides in wireless communications. A SIMO system with a single antenna element at the transmitter and $N$ antenna elements at the receiver has ($N - 1$) DOF that mitigates effectively ($N - 1$) interferences. Similarly to a MISO system with $M$ antenna elements at the transmitter and a single antenna element at the receiver has a capability to cancel ($M - 1$) interferences. As a result, the DOF or maximum cancellation number of interferences for the MIMO system with $M$ antenna elements at the transmitter and $N$ antenna elements at the receiver should be a certain, which is whether larger than that of both SIMO and MISO systems or not, is described here.

In Chapter 4, we propose the weight vectors determination for the transmitter and receiver for MIMO beamforming for a single data stream transmission in frequency-selective fading channels. Through studying the optimal transmit and receive weight vectors determination for MIMO frequency-selective fading, I have successfully proposed the cancellation of ($M + N - 2$) delayed channels for the MIMO beamforming scheme, where $M$ and $N$ are antenna elements at base station (BS) and mobile station (MS), respectively. Based on the proposed MIMO beamforming scheme without using TDL can effectively mitigate multipath fading in frequency-selective channels while having reduced computational complexity.
1.3 Structure of the Thesis

The detailed weight vectors determination for multiuser system of a MIMO beamforming for a single data stream transmission in frequency-selective fading channels is presented in Chapter 5. Based on the method proposed in Chapter 4, we analyze a maximum number of interference channels, which could be eliminated in multiuser system, in two cases of the receive weight vectors optimization only and iterative update of both end weight vectors. For the first case, which is characterized by simple scheme, shown good performance for propagation channel when the preceding channel of the desired user has few its own interference channels, while the second case shown better performance but more sophisticated scheme.

Chapter 6 is an extension of our proposed scheme, which presented in Chapter 4, to improve the performance of system by utilizing multiple delayed versions of the transmitted signal. In Chapter 4, when the preceding wave arrives first which is considered as the desired path, and the subsequent waves represent the undesired paths reflected at increasing distance from the receiver, determination of the transmit and the receive weight vectors for MS and BS, respectively, has been studied to apply to MIMO frequency-selective fading channels for maximizing the SINR. In this research, however, in order to improve the SINR by utilizing multiple independent versions delayed signal of the preceding path instead suppressing them at the BS, a spatial-temporal adaptive MIMO beamforming was proposed and compared with detection of the preceding way only.

Finally, Chapter 7 summarizes the main results of this work and concludes this thesis by suggesting a list of open topics for the future research.
Chapter 2

Fundamentals of Adaptive Arrays

This Chapter presents principal concepts of antenna arrays. Array signal model and different types of adaptive beamforming for narrow and wide band signals are illustrated. In particular, criteria for performance optimization and adaptive algorithms are analyzed in searching the optimal weight vector for antenna array. Finally, the benefits of using adaptive arrays in wireless communication system are also discussed.

2.1 Basic Concepts of Adaptive Signal Processing

An antenna array consists of two or more antenna elements that are spatially arranged by a real-time adaptive processor which produce a directional radiation pattern. Some time antenna array is also referred to as adaptive antennas or smart antennas. Antenna array can be arranged in various geometry configurations of which the most popular are linear, circular and planar shown in Figure 2.1.

A linear antenna array consists of antenna elements separated on a straight line by a given distance. If adjacent elements are equally spaced the array is referred to as a uniformly linear array (ULA). If, in addition, the phase \( \alpha_n \) of the feeding current to the \( n \)th antenna element is increased by \( \alpha_n = n\alpha \), where \( \alpha \) is a constant, then the array is referred to as a progressive phase-shift array. If the feeding amplitudes are constant, then the array is called a uniform array. Similarly, if the array elements are arranged in a circular manner as depicted in Figure 2.1(b), then the array is referred to as an uniform circular array (UCA). The circular array produces beams of a wider width than the corresponding linear array when they are the same
2.1 Basic Concepts of Adaptive Signal Processing

Figure 2.1: Different geometry configurations of antenna arrays.

Figure 2.2: Adaptive Array Configuration.

number of elements and the same spacing between them. While both linear and circular arrays can only perform one-dimensional beamforming (horizontal plane), the planar antenna array can be used for two-dimensional (2-D) beamforming (both in vertical and horizontal planes). An example realization of a planar array is depicted in Figure 2.1(c).

The principle of antenna arrays is the same although theirs geometry configurations are different. However, since the analysis and synthesis are simple, the uniformly linear arrays are often used in both study and experiment compared to that of the rest. The detailed mathematics of geometries can be described in [11]. Throughout this work, the uniformly linear array is restricted to our study. Figure
2.2 Uniformly Linear Array

2.2 shows the most basic structure of the linear adaptive array, which is discussed in these section as follows.

2.2 Uniformly Linear Array

Consider an \( N \)-element ULA which is illustrated in Figure 2.3.

![Figure 2.3: Signal model for ULA.](image)

In Figure 2.3, the array elements are equally spaced by a distance \( d \), and a plan wave arrives at the array from a direction \( \theta \) off the array broad side. The angle \( \theta \) is measured from the principle axis of the array and is called the direction-of-arrival (DOA) or angle-of-arrival (AOA) of the received signal. The plane wave front at the first element should propagate through a distance \( d \sin \theta \) to arrive at the second element. If we take the first element as the reference element and the signal at the reference element is \( s(t) \), then the phase delay of the signal at element \( n \) relative to element 1 is \( (n - 1)kd \sin \theta \), where \( k = 2\pi/\lambda \) is a number waves and \( \lambda \) is the wavelength of the carrier. Therefore, the received signal at the \( n \)th element is given by

\[
x_n(t) = s(t)e^{-j\frac{2\pi}{\lambda}(n-1)d \sin \theta}
\]

where \( j = \sqrt{-1} \) is the imaginary.
2.2 Uniformly Linear Array

Let us arrange $x_n(t)$ in a vector form as
\[
x(t) = [x_1(t) \ x_2(t) \ldots \ x_N(t)]^T
\]
(2.2)

and let
\[
a(\theta) = [1 \ e^{-j\frac{2\pi}{\lambda}d\sin\theta} \ldots \ e^{-j\frac{2\pi}{\lambda}(N-1)d\sin\theta}]^T
\]
(2.3)

where $(.)^T$ denotes the transpose operation. Then equation 2.2 may be expressed in vector form as
\[
x(t) = a(\theta)s(t) + n(t)
\]
(2.4)

where the noise vector has been defined as
\[
n(t) = [n_1(t) \ n_2(t) \ldots \ n_N(t)]^T
\]
(2.5)

The vector $x(t)$ is often referred to as the array input data vector and $a(\theta)$ is called the steering vector. In ULA, the steering vector is only a function of the angle-of-arrival. However, in general, the steering vector is also a function of the individual element response, the array geometry and signal frequency over which the collection of steering vectors for all angles and frequencies is referred to as the array manifold.

It should be noted that if the bandwidth of the impinging signal expressed in (2.1) is much smaller than the reciprocal of the propagation time across the array, the signal is referred to as narrowband signal; otherwise it is referred to as wideband signal.

We now extend the ULA model to a more general case with effects of multipath fading and multiuser. Suppose there are $U$ users impinging on the array and the incident signal of the $i$th user $s_i(t)$ having $L_i$ multipaths are uncorrelated with complex amplitudes $\alpha_{i,l}$, AOA $\theta_{i,l}$ and the excess path delay $\tau_{i,l}$. Then the received signal for the $i$th user may be expressed as
\[
x_i(t) = \sum_{l=1}^{L_i} \alpha_{i,l}a(\theta_{i,l})s_{i,l}(t - \tau_{i,l}) + n(t)
\]
(2.6)

The received signal at the array is a superposition of all the impinging signals and noise. Therefore, the received data vector can be expressed as
\[
x(t) = \sum_{i=1}^{U} \sum_{l=1}^{L_i} \alpha_{i,l}a(\theta_{i,l})s_{i,l}(t - \tau_{i,l}) + n(t)
\]
(2.7)
In matrix notation, (2.7) becomes

\[ x(t) = A(\Theta)s(t) + n(t) \]  \hspace{1cm} (2.8)

where \( A(\Theta) \) is \( N \times U \) matrix of the steering vectors

\[ A(\Theta) = [a(\Theta_1) \ a(\Theta_2) \ ... \ a(\Theta_U)] \]  \hspace{1cm} (2.9)

and

\[ s(t) = [s_1(t) \ s_2(t) \ ... \ s_U(t)]^T \]  \hspace{1cm} (2.10)

### 2.3 Beamforming and Spatial Filtering

Beamforming techniques exist that can yield multiple, simultaneously available beams. The beams can be made to have high gain and low side-lobes, or controlled beam width. Adaptive beamforming techniques dynamically adjust the array pattern to optimize some characteristic of the received signal. The desired and interfering signals usually originate from different spatial locations, antenna arrays can exploit the spatial characteristic to reject interfering signals having a DOA different from that of a desired signal sources. Multi-polarized array can also reject interfering signals having different polarization states from the desired signal, even if the signals have the same DOA. This process of maximizing the signal-to-noise and interference (SINR) based on their spatial characteristic is called spatial filtering and is used in the reverse or uplink (from mobile to base station). Similarly, based on the information estimated from uplink, beamforming is utilized in the forward or downlink (from base station to mobile) to maximize the transmit power of base station to a desired user while suppressing the others. On one hand, antenna arrays using adaptive beamforming are to maximize the SINR, they are called adaptive beamforming. On the other hand, they are designed to steer toward the beams and away from the specific interference locations, it is called null-beamforming.

The antenna elements in an adaptive array collect spatial samples of propagating wave fields, which are processed by the beamformer [12]. Currently, there are two types of beamformer, namely, narrowband beamformer and wideband beamformer. A typical narrowband and wideband beamformers are shown in Figure 2.4 and Figure 2.5.
2.3 Beamforming and Spatial Filtering

In Figure 2.4, the output of array multiplied by a weight vector of the received signal at time $t$ is given by

$$y(t) = \sum_{n=1}^{N} w_n^* x_n(t)$$  \hspace{1cm} (2.11)

where $w_n$ is called the complex weight and $x_n(t)$ is the received signal at sensor $n$th.

Equation (2.11) may also be rewritten in vector form as

$$y(t) = w^H x(t)$$  \hspace{1cm} (2.12)

where $(\cdot)^H$ denotes the complex conjugate (Hermitian) transpose and $w$ is called the complex weight vector and is defined as

$$w = [w_1 \ w_2 \ ... \ w_N]^T$$  \hspace{1cm} (2.13)

In Figure 2.5, the received signal vector is fed into both spatial and temporal domains and employed to process when signals of significant frequency extent (broadband) are of interest. This beamformer process is called spatial-temporal equalizer. The output may be expressed as

$$y(t) = \sum_{n=1}^{N} \sum_{k=0}^{K-1} w_{n,k}^* x_n(t - k)$$  \hspace{1cm} (2.14)
2.3 Beamforming and Spatial Filtering

where \( K - 1 \) is the number of delays in each of the \( N \) sensor elements. Let

\[
\mathbf{w} = [w_{1,0} \ w_{1,1} \ \ldots \ w_{1,K-1} \ \ldots \ w_{N,0} \ w_{N,1} \ \ldots \ w_{N,K-1}]^T \tag{2.15}
\]

and

\[
\mathbf{x}(t) = [x_1(t) \ \ldots \ x_1(t-K+1) \ \ldots \ x_N(t) \ \ldots \ x_N(t-K+1)]^T \tag{2.16}
\]

The output of the broadband beamformer can be now rewritten as

\[
y(t) = \mathbf{w}^H \mathbf{x}(t) \tag{2.17}
\]

The TDL is not only useful for providing the desired adjustment of gain and phase for wideband signals but also for other purpose such as mitigation of multipath under frequency-selective fading and compensation for effects of interchannel mismatch [10, 13]. However, the broadband beamformer using TDL structure is practical difficulties associated with the equalization at several megabits per second with high speed, compact, and low-cost hardware. Beside the broadband beamformer using TDL considered so far is a classical time domain processor, Fast Fourier

Figure 2.5: Configuration of an adaptive wideband beamformer.
transform (FFT) has been used to replace TDL structure in the beamformer configuration resulting in an equivalent frequency domain broadband beamformer as shown in Figure 2.6 [8],[10].

In Figure 2.6, broadband signals from each element are transformed into frequency domain using the FFT and each frequency bin is processed by a narrow-band processor structure. In other words, each signal $x_n(t)$ is decomposed into $K$ subband signals and converted into the frequency domain using the FFT filter bank. After being multiplied by the optimum weights, which estimated from adaptive algorithm such as the Least Mean Square (LMS) algorithm, the weight samples are combined corresponding to each subband. The combined samples are then converted back into time domain using the IFFT filter bank. Finally, the array output signal $y(t)$ is achieved after the parallel-to-serial conversion. The advantage of the frequency domain approach is reduction of computational load and increases the convergence rate. Since the weight vector for each subband is estimated independently, the process of selecting the weight vectors can be performed in parallel, leading to fast weight update [8].
2.4 Adaptive Criteria

With an adaptive array, the signals received by each antenna are weighted and combined to improve output signal performance. Some of the most frequently used performance criteria include Minimum Mean Square Error (MMSE), Maximum-Signal-to-Noise and Interferences Ratio (MSINR), Minimum Variance (MV) and Maximum Likelihood (ML), which will be described below.

2.4.1 Minimum Mean Square Error (MMSE)

The MMSE criterion is first considered by Widrow et al. in [1]. The Criterion make a bid for minimizing the mean-squared error between the desired signal $s(t)$ and the array output $y(t)$ based on a training signal $d(t)$, which is known to both the BS and MS. The training signal $d(t)$ is usually sent from the BS to MS to estimate the propagation environment. After the training period, the data is sent and the information obtained from the training period is useful to process the received data. The training signal is referred to as the reference signal that closely approximates the desired signal. We shall now consider the adaptive array shown in Figure 2.7, the input signal vector is given by

$$x(t) = a(\theta)s(t) + n(t)$$

(2.18)
where \( \mathbf{n}(t) \) is the i.i.d additive noise vector, which is assumed to be complex Gaussian process with zero-mean and variance \( N_0 \), \( \theta \) is the AOA, and \( \mathbf{a} \) is the array propagation vector for the desired signal (steering vector)

\[
\mathbf{a}(\theta) = [1 \ e^{j\pi \sin \theta} \ldots \ e^{j(N-1)\pi \sin \theta}]^T
\]  

(2.19)

The beamformer in the receiver uses the information of the training signal to compute the optimal weight vector \( \mathbf{w}^{(opt)} \). If the channel environment and the interference characteristics remain constant from one training period until the next, the weight vector \( \mathbf{w}^{(opt)} \) will use to give the output \( y(t) \)

\[
y(t) = \mathbf{w}^H \mathbf{x}(t)
\]  

(2.20)

Then the error signal is given by

\[
e(t) = d(t) - y(t) = d(t) - \mathbf{w}^H \mathbf{x}(t)
\]  

(2.21)

and the mean-squared error is defined by

\[
E\{|e(t)|^2\} = E\{|d(t) - \mathbf{w}^H \mathbf{x}(t)|^2\}
\]  

(2.22)

where \( E\{.\} \) denotes the ensemble expectation operator. Expanding (2.22) we have

\[
E\{|e(t)|^2\} = E\{|d(t)|^2\} - \mathbf{w}^T E\{\mathbf{w}^*(t)d(t)\} - \mathbf{w}^H E\{\mathbf{x}(t)d^*(t)\} + \mathbf{w}^H E\{\mathbf{x}(t)\mathbf{x}^H(t)\}\mathbf{w}
\]  

\[
= E\{|d(t)|^2\} - \mathbf{w}^T \mathbf{p}_{xr} - \mathbf{w}^H \mathbf{p}_{xr} + \mathbf{w}^H \mathbf{R}_{xx} \mathbf{w}
\]  

(2.23)

where \( \mathbf{p}_{xr} = E\{\mathbf{x}(t)d^*(t)\} \) is the \( N \times 1 \) cross-correlation vector and \( \mathbf{R}_{xx} = E\{\mathbf{x}(t)\mathbf{x}^H(t)\} \) is the \( M \times M \) correlation matrix. Here (.)\(^*\) denotes the complex conjugate. The optimum weight vector can be found by setting the gradient of (2.23) with respect to \( \mathbf{w} \) equal to zero

\[
\nabla_{\mathbf{w}} E\{|e(t)|^2\} = -2\mathbf{p}_{xr} + 2\mathbf{R}_{xx} \mathbf{w} = 0
\]  

(2.24)

Rearranging

\[
\mathbf{R}_{xx} \mathbf{w} = \mathbf{p}_{xr}
\]  

(2.25)

Assuming \( \mathbf{R}_{xx} \) is non-singular, the optimum solution is found as

\[
\mathbf{w}^{opt} = \mathbf{R}^{-1} \mathbf{p}_{rx}
\]  

(2.26)
2.4 Adaptive Criteria

Equation (2.26) is called the Wiener-Hopf equation [14]. By substituting (2.26) into (2.23), we have the minimum of mean-squared error

$$ E\{|e(t)|^2\} = E\{|d(t)|^2\} - p_x^H R_x^{-1} p_x $$

(2.27)

2.4.2 Maximum-Signal-to-Noise and Interference Ratio (MSINR)

This criterion is used to maximize the output SINR at BS. Starting from (2.18), the output of the array can be expressed as

$$ y(t) = w^H x(t) = w^H s(t) + w^H n(t) $$

$$ = y_s(t) + y_n(t) $$

(2.28)

The average output SINR is given by

$$ \text{SINR} = E\left\{\frac{|y_s(t)|^2}{|y_n(t)|^2}\right\} = E\left\{\frac{w^H s(t) s^H(t) w}{w^H n(t) n^H(t) w}\right\} = \frac{w^H R_{ss} w}{w^H R_{nn} w} $$

(2.29)

where $R_{ss} = s(t)s^H(t)$ and $R_{nn} = n(t)n^H(t)$. Taking the gradient of (2.29) with respect to $w$ is

$$ \nabla_w \text{SINR} = \frac{\nabla_w (w^H R_{ss} w)(w^H R_{nn} w) - (w^H R_{ss} w) \nabla_w (w^H R_{nn} w)}{(w^H R_{nn} w)^2} $$

$$ = \frac{2R_{ss} w(w^H R_{nn} w) - 2R_{nn} w(w^H R_{ss} w)}{(w^H R_{nn} w)^2} $$

(2.30)

The optimum weight vector $w^{\text{opt}}$ can be found by setting $\nabla_w \text{SINR} = 0$, which leads to

$$ R_{ss} = \frac{w^H R_{ss} w}{w^H R_{nn} w} R_{nn} w $$

(2.31)

Noting that $R_{ss} = E\{|s(t)|^2 s^H(t)\} = E\{|s(t)|^2 a(\theta)a^H(\theta)\}$ and $a^H(\theta)w E\{|s(t)|^2\}$ is a scalar, we have

$$ a(\theta) = \frac{w^H a(\theta)}{w^H R_{nn} w} R_{nn} w $$

(2.32)

Define

$$ \frac{1}{\zeta} = \frac{w^H a(\theta)}{w^H R_{nn} w} $$

(2.33)

Then the optimum weight vector can be expressed in a similar form of the Wiener-Hopf equation as

$$ w^{\text{opt}} = \zeta R_{nn}^{-1} a(\theta) $$

(2.34)
2.4 Adaptive Criteria

2.4.3 Minimum Variance (MV)

If the desired signal and its direction are both known, one way of ensuring a good signal reception is to minimize the output noise variance. Minimum variance is also known as linear constrained minimum variance (LCMV). Recall the beamformer output from (2.12)

\[ y(t) = w^H x(t) = w^H a(\theta)s(t) + w^H n(t) \]  \hspace{1cm} (2.35)

In order to ensure that the desired signal is passed with a specific gain and phase, a constraint may be used to so that response of the beamformer to the desired signal is

\[ w^H a(\theta) = g \]  \hspace{1cm} (2.36)

Minimization of contributions of the output due to interference is accomplished by choosing the weight vector to minimize the variance of the output power

\[ \text{Var}\{y(t)\} = w^H R_{ss} w + w^H R_{nn} w \]  \hspace{1cm} (2.37)

subject to the constraint defined in (2.36). Using the method of Lagrange, we have

\[ \nabla_w \left( \frac{1}{2} w^H R_{nn} w + \beta [1 - w^H a(\theta)] \right) = R_{nn} w - \beta a(\theta) \]  \hspace{1cm} (2.38)

where

\[ \beta = \frac{g}{a^H(\theta) R_{nn}^{-1} a(\theta)} \]  \hspace{1cm} (2.39)

then the optimum weight vector using MV criterion can be expressed as

\[ w^{opt} = \beta R_{nn}^{-1} a(\theta) \]  \hspace{1cm} (2.40)

when \( g = 1 \), the MV beamformer is often referred to as the Capon beamformer [14].

2.4.4 Maximum Likelihood (ML)

The Maximum-Likelihood criterion is known to be a powerful approach and frequently used in signal processing. Recall the input signal vector from (2.12)

\[ x(t) = a(\theta)s(t) + n(t) = s(t) + n(t) \]  \hspace{1cm} (2.41)
Letting $p_{x(t)|s(t)}(x(t))$ denotes the probability density function for $s(t)$ given $x(t)$. Since the natural logarithm is a monotone function, we define the Likelihood function as

$$\mathcal{L}(x(t)) = -\ln(p_{x(t)|s(t)}(x(t)))$$

(2.42)

Assume that the $u(t)$ is a stationary zero-mean Gaussian vector having a covariance matrix $R_{uu}$. The Likelihood function can be expressed as

$$\mathcal{L}(x(t)) = C(x(t) - a(\theta)s(t))^HR_{uu}^{-1}(x(t) - a(\theta)s(t))$$

(2.43)

where $C$ is a constant with respect to $x(t)$ and $s(t)$.

The Maximum Likelihood estimate $\hat{s}(t)$ of the $s(t)$ is given by the location of the maximum of the Likelihood function. Using derivatives, the calculation of the Maximum Likelihood estimate becomes

$$\frac{\partial \mathcal{L}(x(t))}{\partial \hat{s}(t)} = -2a^H(\theta)R_{uu}^{-1}x(t) + 2\hat{s}(t)a^H(\theta)R_{uu}^{-1}a(\theta) = 0$$

(2.44)

Since $a^H(\theta)R_{uu}^{-1}a(\theta)$ is a scalar, (2.44) is expressed as

$$\hat{s}(t) = \frac{a^H(\theta)R_{uu}^{-1}x}{a^H(\theta)R_{uu}^{-1}a(\theta)}$$

(2.45)

Comparing (2.45) with (2.12), the optimal weight vector using ML criterion is given by

$$w_{ML}^{(opt)} = \frac{R_{uu}^{-1}a(\theta)}{a^H(\theta)R_{uu}^{-1}a(\theta)}$$

(2.46)

Define

$$\eta = \frac{1}{a^H(\theta)R_{uu}^{-1}a(\theta)}$$

(2.47)

then the optimal weight vector using ML criterion can be expressed

$$w_{ML}^{(opt)} = \eta R_{uu}^{-1}a(\theta)$$

(2.48)

The ML beamformer is also referred to as the Capon beamformer.

### 2.5 Adaptive Algorithms

In the preceding section, we have shown that the optimum criteria are closely related to each other. Therefore, the choice of a particular criterion is not critically important in terms of performance. On the other hand, the choice of adaptive algorithms
for deriving the adaptive weight vector is highly important in that it determines both the speed of convergence and hardware complexity required to implement the algorithm. In this section, we will discuss a number of common adaptive techniques.

2.5 Adaptive Algorithms

The Least Mean Square is the most popular adaptive algorithm for continuous adaptation. It has been well studied and is well understood [15]. It is based on the steepest-descent method, which is proceeded as follows

1. Begin with an initial value $w(0)$ for the weight vector, which is chosen arbitrarily. Typically, $w(0)$ is set equal to a column vector of an $M \times M$ identity matrix.

2. Using this initial or present guess, compute the gradient vector $\nabla(J(t))$ at time $t$ (i.e., the $t$th iteration).

3. Compute the next guess at the weight vector by making a change in the initial or present guess in a direction opposite to that of the gradient vector.

4. Go back to step 2 repeat the process.

It is intuitively reasonable that successive corrections to the weight vector in the direction of the negative of the gradient vector eventually lead to the MSE, at which point the weight vector assumes its optimum value. According to the method of steepest descent, the updated value of the weight vector at time $t+1$ is computed by using the simple recursive relation.

Now, we can describe the LMS algorithm by the following three equations

\[ y(t) = w^H(t)x(t) \quad (2.49) \]

\[ e(t) = d(t) - y(t) \quad (2.50) \]

\[ w(t+1) = w(t) - \mu \nabla E\{e^2(t)\} \quad (2.51) \]

where $\mu$ is the step size which controls the convergence characteristics of $w(t)$

\[ 0 < \mu < \frac{1}{\lambda_{\max}} \quad (2.52) \]
2.5 Adaptive Algorithms

Here $\lambda_{max}$ is the largest eigenvalue of the covariance matrix $R_{xx}$, we have

$$\nabla E\{e^2(t)\} = -2p_{xw} + 2R_{xx}w(t)$$

(2.53)

Replacing (2.53) into (2.41), we have

$$w(t+1) = w(t) + \mu[p_{xw} - R_{xx}w(t)]$$

$$= w(t) + \mu x(t)e^*(t)$$

(2.54)

The LMS algorithm is a member of a family of stochastic gradient algorithms since the instantaneous estimate of the gradient vector is a random vector that depend on the input data vector $x$. The LMS algorithm requires about $2N$ complex multiplications per iteration, where $N$ is the number of weight elements used in the adaptive array. An example of the LMS convergence characteristic is shown in Figure 2.8 More details about the LMS algorithm are discussed in [16],[17].

![Figure 2.8: An example of the LMS learning curve using linear array elements with $d = \lambda/2$, $N = 4$, $\mu = 0.005$, $\text{SNR}_{in} = 10\text{dB}$.](image)

2.5.2 Sample Matrix Inversion (SMI)

$$R_{xx}(t) = \frac{1}{t} \sum_{i=1}^{t} x(i)x^H(i)$$

(2.55)
\[ r_{xx}(t) = \frac{1}{t} \sum_{i=1}^{t} x(i)p^*(i) \] (2.56)

It follows that the estimated weight vector using the SMI algorithm is given by

\[ w(t) = R_{xx}^{-1}(t)p_{xx}(t) \] (2.57)

Note that the SMI is a block-adaptive algorithm and has been shown to be the fastest algorithm for estimating the optimum weight vector [18]. However, it suffers the problems of increased computational complexity and numerical instability due to inversion of a large matrix.

### 2.5.3 Recursive Least Square (RLS)

Unlike the LMS algorithm which uses the method of steepest-descent to update the weight vector, the Recursive Least Square (RLS) algorithm uses the method of least-squares to adjust the weight vector. In the method of least squares, we choose the weight vector \( w(t) \), so as to minimize a cost function that consists of the sum of error squares over a time window. In the method of steepest-descent, on the other hand, we choose the weight vector to minimize the ensemble average of the error squares.

In the exponentially weighted RLS algorithm, at time \( t \), the weight vector is chosen to minimize the cost function.

\[ Q(t) = \sum_{i=1}^{t} \gamma^{-i} |e(i)|^2 \] (2.58)

where \( \gamma \) is a positive constant close to one, which determines how quickly the previous data are de-emphasized. In a stationary environment, however, \( \gamma \) should be equal to 1, since all data past and present should have equal weight. The RLS algorithm can be described by the following equations

\[ q(t) = \frac{\gamma^{-1}P(t-1)x(t)}{1 + \gamma^{-1}x^H(t)P(t-1)x(t)} \] (2.59)

\[ \alpha(t) = d(t) - w^H(t-1)x(t) \] (2.60)

\[ w(t) = w(t-1) + q(t)\alpha^*(t) \] (2.61)
2.5 Adaptive Algorithms

\[
P(t) = \gamma^{-1}P(t-1) - \gamma^{-1}q(t)w^H(t)P(t-1)
\]  

(2.62)

The initial value of \( P(t) \) can be set to

\[
P(0) = \delta^{-1}I
\]  

(2.63)

where \( I \) is the \( N \times N \) identity matrix, and \( \delta \) is a small positive constant.

An important feature of the RLS algorithm is that it utilizes information contained in the input data, extending back to the instant of time when the algorithm is initiated. An example of the convergence characteristic of the RLS algorithm is depicted in Figure 2.9. The resulting rate of convergence is therefore typically an order of magnitude faster than the simple LMS algorithm. This improvement in performance, however, is achieved at the expense of a large in computational complexity. The RLS algorithm requires \((4N^2 + 4N + 2)\) complex multiplications per iteration, where \( N \) is the number of weights used in the adaptive array. More details about the RLS algorithm are discussed in \([19],[20]\).

![Figure 2.9: An example of the RLS learning curve using linear array elements with \( d = \lambda/2, N = 4, \gamma = 1, \text{SNR}_{\text{in}} = 10\text{dB} \)]
2.6 Benefits of Using Adaptive Arrays in Wireless Communication Systems

If a base station in a cellular system uses an adaptive, several benefits are produced [21, 22]:

## 2.6.1 Signal Quality Improvement

The antenna gain is the increased average output SINR with these multiple antennas. Define the input SNR as $\text{SNR}_{\text{in}}$ than if the $N$ antennas are employed, the combined signals are added in phase, while the noise is added incoherently, producing $(N-1)$ degree of freedom to suppress $(N-1)$ interferences. In a propagation environment without multipath fading, the output SINR can be found as

$$\text{SINR}_{\text{out}} = N \times \text{SNR}_{\text{input}}$$  \hspace{1cm} (2.64)

or

$$\text{SINR}_{\text{out}}[\text{dB}] = \log_{10} N + \text{SNR}_{\text{input}}[\text{dB}]$$  \hspace{1cm} (2.65)

From (2.65), it is clear that the array gain achieved by an adaptive array is

$$G = \log_{10} N$$  \hspace{1cm} (2.66)

In the multipath fading environment, if $L$ delayed versions of the transmitted signal are exploited effectively, the output SINR is given by

$$\text{SINR}_{\text{out}}[\text{dB}] = G + 10 \log_{10}(L) + \text{SNR}_{\text{in}}[\text{dB}]$$  \hspace{1cm} (2.67)

Let us taking a simple case of spatially uncorrelated 2-paths model as an example, the output SINR is estimated as

$$\text{SINR}_{\text{out}}[\text{dB}] = G + 10 \log_{10}(2) + \text{SNR}_{\text{in}}[\text{dB}]$$  \hspace{1cm} (2.68)

Figure 2.10 shows the SNR versus the number of employed array elements. This means that the richer the multipath fading environment is, the more diversity gain can be achieved.
2.6 Benefits of Using Adaptive Arrays in Wireless Communication Systems

2.6.2 Range Extension

An important benefit of smart antennas is range extension. Range extension allows the mobile to operate farther from the base station without increasing the uplink power transmitted by the mobile unit or the downlink power required from the base station transmitter.

For a constant path loss exponent of $l \geq 2$, the range of a cell using adaptive array $R_a$ is greater than the range using conventional antenna $R_c$. The range extension factor (REF) is given by [23]

$$REF = \frac{R_a}{R_c} = M^{1/l} \quad (2.69)$$

Then the extended area coverage factor (ECF), which is the ratio of the area of a cell covered with adaptive array $A_a$ to the area of a cell covered using conventional antenna $A_c$, is given by

$$ECF = \frac{A_a}{A_c} = \left(\frac{R_a}{R_c}\right)^2 = M^{2/l} \quad (2.70)$$

Figure 2.10: Output SNR versus number of array elements.
2.6.3 Increase in Capacity

Capacity is related to the spectral efficiency of a system. The spectral efficiency $E$ measured in channels/km$^2$/MHz is expressed as

$$E = \frac{B_t}{B_t N_c A_c} = \frac{1}{B_{ch} N_c A_c}$$  \hspace{1cm} (2.71)

where $B_t$ is the total bandwidth of the system available for voice channels in MHz, $B_{ch}$ is the bandwidth per voice channel in MHz, $N_c$ is the number of cells per cluster. The capacity of a system is measured in channels/km$^2$ and is given by [24],[25]

$$C = E B_t = \frac{B_t}{B_{ch} N_c A_c} = \frac{N_{ch}}{N_c A_c}$$  \hspace{1cm} (2.72)

where $N_{ch} = B_t / B_{ch}$ is the total number of available voice channels in the system.

Example 1

A system with $N_{ch} = 280$ channels and with conventional base station antennas uses a seven-cell frequency reuse pattern ($N_c = 7$). Each cell covers an area of $A_c = 50$km$^2$. From (2.72), the capacity is $C_c = \frac{280}{7 \times 50} = 0.8$ channels/km$^2$. By using an adaptive array at the base station, ICI is reduced and $N_c$ can be reduced to 4. The capacity is $C_a = \frac{280}{4 \times 50} = 1.4$ channels/km$^2$.

It is clear that use of adaptive array can improve system capacity compared to the conventional system at the same range.

2.6.4 Reduction in Transmit Power

Based on the array gain achieved by an adaptive array, the reduction in the required transmit power of the base station is available. Consequently, on the one hand, the reduction in the transmit power is beneficial to user’s health. On the other hand, the battery life can be extended.

2.7 Summary

We have provided an overview of adaptive arrays for wireless communications. The array signal models of the narrowband and broadband beamforming for multipath
fading environments were described. Then the essential features of four criteria, namely, MMSE, MSINR, MV, ML, and adaptive algorithms, namely, LMS, SMI, RSL for finding optimal weight vector of adaptive arrays were illustrated. Finally, we have shown that using adaptive arrays at base station can bring several efficient benefits for wireless communication systems.
Chapter 3

Multiple-Input Multiple-Output (MIMO) System for Wireless Communications

3.1 Basic Concepts

The application of adaptive antennas to mobile systems has significant advantages in terms of coverage, channel capacity and signal quality. Several adaptive antenna systems have been proposed and presented at the base station (BS) of the wireless communication systems. However, demands for broadband wireless access technologies such as mobile internet, multi-media services given by wireless communication systems are rapidly growing during the latest few years. In the effort to deliver high-bit-rates in broadband wireless systems, the transmission techniques are required to be able to cope with multipath fading channels. One of the most expected solutions is using adaptive antennas at both transmitter and receiver [26]-[39]. It is referred to as a multiple-input multiple-output (MIMO) antennas system. MIMO systems are classified into two groups: (1) research on a high-quality transmission of a single data stream such as space-time codes, a transmit/receive diversity, and transceiver beamforming [30]-[37] or (2) research on a high-data-rate transmission of multiple independent data streams such as VBLAST, MIMO-OFDM [38]-[42].

Although a single data stream or multiple data streams is propagated and mixed in the air, they can be recover at the receiver by using spatial filter and corresponding signal processing. This Chapter presents the principle of a single data and multiple data streams transmissions of a MIMO system under flat and frequency-selective fading channels with the perfect Channel State Information (CSI) at both sides. Finally, motivation to use of a MIMO beamforming method in multipath
fading environment is discussed.

3.2 Narrowband MIMO Channel

3.2.1 Single Data Stream Transmission

A narrowband communication system of $M$ transmit and $N$ receive antennas is shown in Figure 3.1.

$$y(t) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} w^*_r n_{r,n} + \sum_{n=0}^{N-1} w^*_r n_n(t)$$ (3.1)

The equation (3.1) can be expressed in a vector form as

$$y(t) = w_t^H A w_t s(t) + w_r^H n(t)$$ (3.2)
3.2 Narrowband MIMO Channel

where
\[
\mathbf{w}_r = [w_{r,0}, w_{r,1}, \ldots, w_{r,N-1}]^T \tag{3.3}
\]
and
\[
\mathbf{w}_t = [w_{t,0}, w_{t,1}, \ldots, w_{t,M-1}]^T \tag{3.4}
\]
Here \((.)^H\), \((.)^*\) and \((.)^T\) represent the Hermitian transpose, conjugate and transpose of vector (or matrices).

Based on the maximal ratio combining (MRC) method, with the transmit and receive weight vectors \(\mathbf{w}_t\) and \(\mathbf{w}_r\) in terms of the constraint of \(\|\mathbf{w}_t\| = \|\mathbf{w}_r\| = 1\),
the maximum of output signal-to-noise ratio is given by
\[
\text{SNR}_{\text{out}} = \frac{\mathbf{w}_t^H \mathbf{A} \mathbf{w}_t \mathbf{A}^H \mathbf{w}_r}{\|\mathbf{w}_r\| \mathbf{P}_\text{S}} \mathbf{P}_\text{N} \tag{3.5}
\]
where \(\mathbf{P}_\text{S}\) and \(\mathbf{P}_\text{N}\) are power of the transmitted signal and the noise. Thus, \(\mathbf{P}_\text{S}/\mathbf{P}_\text{N}\)
is referred to the input SNR.

If the channel matrix \(\mathbf{A}\) is known well at both sides, the received SNR is optimized by choosing the weight vectors \(\mathbf{w}_r\) and \(\mathbf{w}_t\) as the principal left and right singular vectors of the channel matrix \(\mathbf{A}\). The corresponding received SNR is given by
\[
\text{SNR}_{\text{out}} = \lambda_{\text{max}} \text{SNR}_\text{in} \tag{3.6}
\]
where \(\lambda_{\text{max}}\) is the largest eigenvalue of the Wishart matrix \(\mathbf{A} \mathbf{A}^H\).

The resulting capacity can be given by [38],[43]
\[
C = \log_2(1 + \lambda_{\text{max}} \text{SNR}_\text{in}) \quad \text{b/s/Hz} \tag{3.7}
\]

3.2.2 Multiple Data Streams Transmission

A narrowband communication system of \(M\) transmit and \(N\) receive antennas for multiple data streams transmission is shown in Figure 3.2.

From the singular value decomposition (SVD) theory, we have
\[
\mathbf{A} = \mathbf{w}_r \Sigma \mathbf{w}_t = \sum_{p=1}^{P} \sqrt{\lambda_p} \mathbf{w}_{r,p} \mathbf{w}_{t,p}^H \tag{3.8}
\]
3.2 Narrowband MIMO Channel

where \( \mathbf{w}_r \) and \( \mathbf{w}_t \) are unitary matrices of left and right singular vector

\[
\mathbf{w}_r = [\mathbf{w}_{r,1}, \mathbf{w}_{r,2}, ..., \mathbf{w}_{r,P}]
\]

\[
\mathbf{w}_t = [\mathbf{w}_{t,1}, \mathbf{w}_{t,2}, ..., \mathbf{w}_{t,P}]
\]

and \( \Sigma \) is a diagonal matrix of singular values

\[
\Sigma = \text{diag}(\lambda_1^{1/2}, \lambda_2^{1/2}, ..., \lambda_P^{1/2})
\]

where

\[
\lambda_1 > \lambda_2 > ... > \lambda_P > 0
\]

and \( P \) is the rank of the channel matrix

\[
P = \text{rank}(\mathbf{A}) = \min(M, N)
\]

The SVD transforms the MIMO channel into \( P \) parallel independent channels and the performance on each of the channels will be depend on its gain \( \lambda_p \).

The resulting capacity can be expression as [38],[43]

\[
C = \sum_{p=1}^{P} \log_2(1 + \lambda_p \text{SNR}_{\text{in}}) \quad \text{b/s/Hz}
\]
3.3 Wideband MIMO Channel

In case of flat fading, the beamforming or SVD with power allocation scheme by water filling is known to be an effective approach under the assumption of perfect CSI in both the transmitter and the receiver. And besides, various space-time coding techniques are used when the transmitter or the receiver is uniformed [38],[44]. However, under frequency-selective multipath fading channels, those methods could not be simply applied due to the inter symbol interference (ISI) caused by signals arriving through delay paths. Recently, two architectures have been investigated for MIMO systems to mitigate the effect of frequency-selective fading channels. The first architecture is transmission of multiple data streams through spatial multiplexing or space-time codes combined with the orthogonal frequency division multiplexing. However, the multicarrier MIMO system is considered a most attractive candidate since the frequency-selective MIMO channels are transformed into several frequency flat MIMO sub-channel by using OFDM system [45]-[47]. Besides multicarrier MIMO system using either the spatial multiplexing OFDM or space-time code OFDM, the second architecture is a single carrier MIMO system using beamforming, which has been studied extensively [48]-[51]. Throughout this work, we shall restrict our study to the single carrier MIMO system under frequency-selective multipath fading environment with prior knowledge of CSI at both the transmitter and the receiver.

3.3.1 Single Data Stream Transmission

A general MIMO beamforming for the single data stream transmission under frequency-selective fading channels is shown in Figure 3.3. The number of transmit and receive antennas is $M$ and $N$, respectively.

For wireless broadband systems, the MIMO propagation channel can be modeled as

$$H(\tau) = \sum_{l=0}^{L} A(l) \delta(\tau - l\triangle\tau)$$  \hspace{1cm} (3.15)
3.3 Wideband MIMO Channel

Figure 3.3: Broadband MIMO channel and beamforming configuration.

\[ \mathbf{A}^{(l)} = \begin{pmatrix} a_{11}^{(l)} & a_{12}^{(l)} & \cdots & a_{1M}^{(l)} \\ a_{21}^{(l)} & a_{22}^{(l)} & \cdots & a_{2M}^{(l)} \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1}^{(l)} & a_{N2}^{(l)} & \cdots & a_{NM}^{(l)} \end{pmatrix} \quad (3.16) \]

\( \mathbf{A}^{(0)} (l=0) \) is considered the channel information of the preceding wave and regarded as the desired channel. \( \mathbf{A}^{(l)} (l = 1, \ldots, L) \) is the \( l \)th delayed channel information which is considered interference channels. The notation \( a_{mn}^{(l)} \) means the \( l \)th delayed path gain response between the \( m \)th transmit antenna and the \( n \)th receive antenna. \( \Delta\tau \) is the unit delay time, which corresponds to symbol period \( T_s \), of the modulated signal.

The output of array elements is combined with a weight vector to recover the transmitted data. Thus the output signal at the receiver is expressed as

\[ y(t) = \sum_{l=0}^{L} \mathbf{w}_r^H \mathbf{A}^{(l)} \mathbf{w}_t s(t - l\Delta\tau) + \mathbf{w}_r^H \mathbf{n}(t) \quad (3.17) \]

where \( s(t) \) is the source signal, and \( \mathbf{n} = [n_1, n_2, \ldots, n_N]^T \) is the additive white Gaussian noise (AWGN) vector. The beamforming transmit and receive weight vectors \( \mathbf{w}_t, \mathbf{w}_r \) are defined as

\[ \mathbf{w}_t = [w_{t1}, w_{t2}, \cdots, w_{tM}]^T \quad (3.18) \]

\[ \mathbf{w}_r = [w_{r1}, w_{r2}, \cdots, w_{rN}]^T \quad (3.19) \]
3.3 Wideband MIMO Channel

Assume that each delayed signal is uncorrelated and zero-mean, thereby

\[ \langle s^*(t - i\Delta\tau)s(t - j\Delta\tau) \rangle = 0 \quad \text{for} \quad i \neq j \quad (3.20) \]

Let us define \( P_s, P_N \) and \( 1/\gamma \), which are the signal power, noise power and power ratio of the signal to noise.

\[ \langle |s|^2 \rangle = P_s \quad (3.21) \]
\[ \langle |n_1|^2 \rangle = \langle |n_2|^2 \rangle \cdots = \langle |n_N|^2 \rangle = P_N \quad (3.22) \]
\[ 1/\gamma \equiv P_s/P_N \quad (3.23) \]

Therefore, the SINR at the receiver is given by

\[ \Gamma(w_t, w_r) = \frac{|w_t^H A(0) w_t|^2 P_s}{|\sum_{l=1}^{L} w_r^H A(l) w_t|^2 P_s + |w_r|^2 P_N} \]
\[ = \frac{w_t^H A(0) w_t w_r^H (A(0))^H w_r}{\sum_{l=1}^{L} w_r^H A(l) w_t w_t^H (A(l))^H w_r + \gamma |w_r|^2 w_r} \quad (3.24) \]

3.3.2 Multiple Data Streams Transmission

A general MIMO beamforming for multiple data stream transmission under frequency-selective fading channels is shown in Figure 3.4. The number of transmit and receive antennas is \( M \) and \( N \), respectively. The received signal at receiver can be expressed as

\[ r(t) = \sum_{i=1}^{Q} \sum_{l=0}^{L} A(l) w_t^{(i)} s^{(i)}(t - l\Delta\tau) + n(t) \quad (3.25) \]

where \( s^{(i)} \) is the \( i \)th transmit data, \( n(t) \) is the i.i.d additive noise, which is assumed to be complex Gaussian process with zero-mean and variance \( N_0/2 \). The beamforming transmit weight vector \( w_t^{(i)} \) is estimated for finding the largest eigenvalue for \( \lambda_{\text{max}} \) of the correlation matrix \( (A^{(i)})^H A^{(i)} \) when \( A^{(i)}(i \leq L) \) is considered as the desired channel.

Assume that the source signals are mutually uncorrelated with zero-mean and unit variance, thereby

\[ \langle (s^{(i)}(t - l\Delta\tau))^* s^{(i)}(t - k\Delta\tau) \rangle > \quad \text{for} \quad l \neq k \quad (3.26) \]
3.3 Wideband MIMO Channel

Figure 3.4: Broadband MIMO channel and beamforming for multiple data streams configuration.

\[
< (s^{(i)}(t))^* s^{(j)}(t) > = 0 \quad \text{for} \quad i \neq j \quad (3.27)
\]

Multiplying complex weight vector \( \mathbf{w}_r^{(k)} \) for detecting the transmitted data stream \( s^{(k)}(t) \), the output signal \( y^{(k)}(t) \) is given by

\[
y^{(k)}(t) = \sum_{i=1}^{Q} \sum_{l=0}^{L} \mathbf{w}_r^{(k)} \mathbf{A}^{(l)} \mathbf{w}_t^{(i)} s^{(i)}(t - (l + k - 1)\Delta \tau) + \mathbf{w}_r^{(k)} \mathbf{n}(t) \quad (3.28)
\]

The output SINR at the BS for finding the transmitted data stream \( s^{(k)}(t) \) is given by

\[
\Gamma(\mathbf{w}_t^{(k)}, \mathbf{w}_r^{(k)}) = \frac{(\mathbf{w}_r^{(k)})^H \mathbf{A}^{(k)} \mathbf{w}_t^{(k)} (\mathbf{w}_t^{(k)})^H (\mathbf{A}^{(k)})^H \mathbf{w}_r^{(k)}}{R_{inf}^{(k)} + \gamma^{(k)} (\mathbf{w}_r^{(k)})^H \mathbf{w}_r^{(k)}} \quad (3.29)
\]

where \( R_{inf}^{(k)} \) is the sum of interferences of the desired signal for the transmitted data stream \( s^{(k)}(t) \) and interferences caused by other transmitted data streams.

\[
R_{inf}^{(k)} = \sum_{l=0, l \neq k}^{L} (\mathbf{w}_r^{(k)})^H \mathbf{A}^{(l)} \mathbf{w}_t^{(k)} (\mathbf{w}_t^{(k)})^H (\mathbf{A}^{(l)})^H \mathbf{w}_r^{(k)}
\]

\[
+ \sum_{i=1, i \neq k}^{Q} \sum_{l=0}^{L} (\mathbf{w}_r^{(i)})^H \mathbf{A}^{(l)} \mathbf{w}_t^{(i)} (\mathbf{w}_t^{(i)})^H (\mathbf{A}^{(l)})^H \mathbf{w}_r^{(i)} \quad (3.30)
\]
3.4 Motivations

As we have mentioned earlier, CCI and ISI are two factors which degrade the performance of wireless communication systems. Adaptive array utilizing a spatial filter has capability to mitigate CCI. For instance, an $N$ element antenna array has $(N - 1)$ DOF and thus can suppress $(N - 1)$ CCIs independent of the multipath environment [18],[22]. However, adaptive array exploited a spatial domain is unavailable to treat the delayed versions of the transmitted signal as separated signals. One of solutions is combination with temporal filter using TDL structure. For demands in wireless access technologies such as mobile quality and high speed service, the needs of the high performance and capacity of wireless communications system are required. The MIMO antenna systems using TDL structure for suppressing the ISI have been absolutely considered. However, the SIMO or MISO systems using TDL structure have been known to be practical difficulties associated with the equalization at several megabits per second with high speed, compact and low cost hardware. Therefore, although MIMO systems using TDL structure have capability to suppress both CCI and ISI [50]-[53], but they are more difficult from computational complexity and low convergence in searching for optimal weights.

On the other hand, Although MIMO systems using TDL structure adapt to mitigate the CCI and ISI in wireless communications, they do not yet point out clearly how many delayed channels can be effectively cancelled by using adaptive beamforming by adjusting both the transmitter and the receiver weight vectors. Since antenna array is used in both the transmitter and the receiver of MIMO systems, the DOF or maximum cancellation number of delayed channels is considered to be larger than that of both SIMO and MISO systems. Through straightforward thinking, we expect that DOF of MIMO systems with $M$ antenna elements at the transmitter antennas and $N$ antenna elements at the receiver ($M \times N$ MIMO systems) is given by

$$\text{DOF} = M + N - 2$$  \hspace{1cm} (3.31)

Assume that the input SNR is the threshold for communicating. In order to achieve the equation (3.31), the output SINR obtained from the equation (3.24) must be larger or equal to that of the input SNR in all cases of number of delayed channels increasing from 1 to $(M + N - 2)$. Since both the transmit and receive
weight vectors, however, are contained in the numerator and denominator, the equation (3.24) becomes a multivariable nonlinear equation. It seems difficult to find optimal transmit and receive weight vectors analytically. Meanwhile, in order to resolve the problem, we propose a solution to find the optimal transmit and receive weight vector based on an iterative weight update algorithm for maximizing the output SINR under the effects of frequency-selective fading channels.

3.5 Summary

Basic overviews of the single and multiple data streams transmission models for both narrow and wide bands MIMO beamforming with the perfect CSI at both sides under multipath fading were presented. It was shown that the optimal transmit and receive weight vectors become one of main factors to suppress CCI and ISI in wireless communications system.

The motivations of our proposed scheme for MIMO beamforming method in wireless communications system were also discussed. It was expected that our propose scheme can effectively mitigate both CCI and ISI while maximizing the output SINR at the receiver. On the other hand, based on our proposed method, the convergence rate in searching the optimal weights and computational complexity are reduced considerably.
Chapter 4

MIMO Beamforming for Single Data Stream Transmission in FSF Channels

4.1 Propagation Model

Consider a MIMO wireless communication system shown in Figure 4.1, where $M$ and $N$ is the number of antenna elements of the transmitter and receiver, respectively. The idea is to transmit a single data stream simultaneously on the different antennas of the transmitter, but at the same carrier frequency.

![Figure 4.1: Broadband MIMO channel and beamforming configuration.](image)

For wireless broadband systems, the MIMO propagation channel can be modeled
4.1 Propagation Model

as

\[
H(\tau) = \sum_{l=0}^{L} A^{(l)}(\tau - l\triangle\tau) \tag{4.1}
\]

\[
A^{(l)} = \begin{pmatrix}
a^{(l)}_{11} & a^{(l)}_{12} & \cdots & a^{(l)}_{1M} \\
a^{(l)}_{21} & a^{(l)}_{22} & \cdots & a^{(l)}_{2M} \\
\vdots & \vdots & \ddots & \vdots \\
a^{(l)}_{N1} & a^{(l)}_{N2} & \cdots & a^{(l)}_{NM}
\end{pmatrix} \tag{4.2}
\]

where \(A^{(0)}(l = 0)\) is the channel information of the preceding wave which we regard as the desired channel. \(A^{(l)}(l = 1, \ldots, L)\) is the \(l\)th delayed channel information which we consider as interference channel. The notation \(a^{(l)}_{nm}\) means the \(l\)th delayed path gain response between the \(m\)th transmit antenna and the \(n\)th receive antenna. \(\triangle\tau\) is the unit delay time, which corresponds to symbol period \(T_s\), of the modulated signal.

Since the received signal is assumed stationary, the output of antenna array is combined with complex weight vector to recover the transmitted signal. Thus the received signal at the receiver is expressed as

\[
y(t) = \sum_{l=0}^{L} w_r^H A^{(l)} w_t s(t - l\triangle\tau) + w_r^H n(t) \tag{4.3}
\]

where \(s(t)\) is the transmitted signal, and \(n = [n_1, n_2, \ldots, n_N]^T\) is the additive white Gaussian noise (AWGN) vector. The beamforming transmit and receive weight vectors \(w_t, w_r\) are defined as

\[
w_t = [w_{t1}, w_{t2}, \cdots, w_{tM}]^T \tag{4.4}
\]

\[
w_r = [w_{r1}, w_{r2}, \cdots, w_{rN}]^T \tag{4.5}
\]

Assume that each delayed signal is uncorrelated and zero-mean, thereby

\[
\langle s^*(t - i\Delta\tau) s(t - j\Delta\tau) \rangle = 0 \quad \text{for} \quad i \neq j \tag{4.6}
\]

The power of the received signal is given by

\[
y^2(t) = |w_r^H A^{(0)} w_t s(t) + \sum_{l=1}^{L} w_r^H [A^{(l)} w_t s(t - l\Delta\tau)] + w_r^H n(t)|^2
\]

\[
= |w_r^H A^{(0)} w_t s(t)|^2 + \left| \sum_{l=1}^{L} w_r^H [A^{(l)} w_t s(t - l\Delta\tau)] \right|^2 + |w_r^H n(t)|^2 \tag{4.7}
\]
4.2 Transmit and Receive Weight Vectors Determination

Let us define $P_s, P_I, P_N$ and $1/\gamma$ the signal power, interference power, noise power and power ratio of the signal to noise. The power of the received signal is rewritten by

$$y^2(t) = |w_r^H A^{(0)} w_t|^2 P_s + \sum_{l=1}^{L} |w_r^H A^{(l)} w_t|^2 P_I^l + |w_r^H w_r|^2 P_N$$

(4.8)

Therefore, the output SINR at the receiver is given by

$$\Gamma(w_t, w_r) = \frac{w_r^H A^{(o)} w_t w_t^H (A^{(o)})^H w_r}{\sum_{l=1}^{L} w_r^H A^{(l)} w_t w_t^H (A^{(l)})^H w_r + \gamma w_r^H w_r} = \frac{w_r^H R_0 w_r}{w_r^H R_{nr} w_r}$$

(4.9)

where

$$R_0 = A^{(o)} w_t w_t^H (A^{(o)})^H$$

(4.10)

and

$$R_{nr} = \sum_{l=1}^{L} w_r^H A^{(l)} w_t w_t^H (A^{(l)})^H w_r + \gamma w_r^H w_r$$

(4.11)

The equation (4.9) is a multivariable nonlinear equation. Since both the transmit and receive weight vectors are contained in numerator and denominator, it seems difficult to find optimal transmit and receive weight vectors analytically. Meanwhile, in order to resolve this, we propose a solution to determine the optimal transmit and receive weight vectors based on an iterative weight update algorithm for maximizing the output SINR under the effects of frequency-selective fading channels as follows.

### 4.2 Transmit and Receive Weight Vectors Determination

#### 4.2.1 Receive Weight Vector Determination

Assume that the transmit weight vector is given with a mandatory condition of $w_t^H w_t = 1$ and an optional condition of $w_r^H w_r = 1$, the steering vector of the desired signal is $A^{(0)} w_t$, thereby, the receive weight vector which are resolved by Maximum-Signal-to-Noise (MSN) method [16], can be expressed as follows.
4.2 Transmit and Receive Weight Vectors Determination

Taking the gradient of equation (4.9) with respect to \( w_r \) give

\[
\nabla \Gamma_{w_r}(w_t, w_r) = \frac{\nabla(w_t^H R_0 w_r)(w_t^H R_{nr} w_r) - (w_t^H R_0 w_r) \nabla(w_t^H R_{nr} w_r)}{(w_t^H R_{nr} w_r)^2}
\]

\[
= \frac{2R_0 w_r (w_t^H R_{nr} w_r) - 2R_{nr} w_r (w_t^H R_0 w_r)}{(w_t^H R_{nr} w_r)^2}
\]

(4.12)

The optimal receive weight vector \( w_r^{(opt)} \) can be found by setting \( \nabla \Gamma_{w_r}(w_t, w_r) = 0 \), which leads to

\[
R_0 w_r = \frac{w_t^H R_0 w_r}{w_t^H R_{nr} w_r} R_{nr} w_r
\]

(4.13)

Substituting (4.10) into (4.13), we have

\[
A^{(o)} w_t w_t^H (A^{(o)})^H w_r = \frac{w_t^H A^{(o)} w_t w_t^H (A^{(o)})^H w_r}{w_t^H R_{nr} w_r} R_{nr} w_r
\]

(4.14)

Since \( w_t^H (A^{(o)})^H w_r \) is a scalar, (4.14) is rewritten as

\[
A^{(o)} w_t = \frac{w_t^H A^{(o)} w_t}{w_t^H R_{nr} w_r} R_{nr} w_r
\]

(4.15)

Let us define \( \frac{1}{\zeta} \equiv \frac{w_t^H A^{(o)} w_t}{w_t^H R_{nr} w_r} \), then the receive weight vector is given by

\[
w_r = \zeta R_{nr}^{-1} A^{(o)} w_t
\]

(4.16)

In equation (4.11), \( R_{nr} \), is an Hermitian matrix, and the coefficient \( \zeta \) is a complex scalar that does not affect the received SINR itself in (4.16). Thus the optimal receive weight vector can be normalized and given by

\[
w_r^{(opt)} = \frac{R_{nr}^{-1} A^{(o)} w_t}{\|R_{nr}^{-1} A^{(o)} w_t\|}
\]

(4.17)

Accordingly, the optimal receive weight vector can be determined if the transmit weight vector is given.

4.2.2 Transmit Weight Vector Determination

Assume that the receive weight vector is given that is allowed to determine the optimal transmit weight vector satisfying the condition \( w_t^H w_t = 1 \). To find the
4.2 Transmit and Receive Weight Vectors Determination

optimal transmit weight vector which maximizes output SINR at the receiver, we use the Lagrange multiplier method [54] as follows.

\[ \phi = \Gamma(\mathbf{w}_t, \mathbf{w}_r) + \lambda(1 - \mathbf{w}_t^H \mathbf{w}_t) \quad (4.18) \]

From equation (4.9), we rewrite \( \Gamma(\mathbf{w}_t, \mathbf{w}_r) \) expression as

\[
\Gamma(\mathbf{w}_t, \mathbf{w}_r) = \frac{\sum_{l=1}^{L} \mathbf{w}_t^H \mathbf{A}^{(l)} \mathbf{w}_t \mathbf{A}^{(l)^H} \mathbf{w}_r + \gamma \mathbf{w}_r^H \mathbf{w}_r}{\sum_{l=1}^{L} \mathbf{w}_t^H \mathbf{A}^{(l)} \mathbf{w}_t} = \frac{\mathbf{w}_t^H \tilde{\mathbf{R}}_0 \mathbf{w}_t + \gamma \mathbf{w}_r^H \mathbf{w}_r}{\mathbf{w}_t^H \mathbf{R}_{lt} \mathbf{w}_t} \quad (4.19)
\]

where

\[ \tilde{\mathbf{R}}_0 \equiv (\mathbf{A}^{(o)})^H \mathbf{w}_t \mathbf{w}_r^H \mathbf{A}^{(o)} \quad (4.20) \]

and

\[ \mathbf{R}_{lt} \equiv \sum_{l=1}^{L} (\mathbf{A}^{(l)})^H \mathbf{w}_t \mathbf{w}_r^H \mathbf{A}^{(l)} \quad (4.21) \]

Taking the derivative of equation (4.18) with respect to \( \mathbf{w}_t \) and setting it to zero.

\[
\frac{\partial \phi}{\partial \mathbf{w}_t} = \frac{\partial \Gamma}{\partial \mathbf{w}_t} + \frac{\partial \lambda(1 - \mathbf{w}_t^H \mathbf{w}_t)}{\partial \mathbf{w}_t} = 0
\]

then,

\[
\tilde{\mathbf{R}}_0 \mathbf{w}_t \left( \mathbf{w}_t^H \mathbf{R}_{lt} \mathbf{w}_t + \gamma \| \mathbf{w}_r \|^2 \right) - \mathbf{R}_{lt} \mathbf{w}_t \left( \mathbf{w}_t^H \tilde{\mathbf{R}}_0 \mathbf{w}_t \right) - \lambda \mathbf{w}_t = 0 \quad (4.22)
\]

By multiplying the left and right sides of (4.23) by weight vector \( \mathbf{w}_t^H \), we have

\[
\mathbf{w}_t^H \tilde{\mathbf{R}}_0 \mathbf{w}_t \left( \mathbf{w}_t^H \mathbf{R}_{lt} \mathbf{w}_t + \gamma \| \mathbf{w}_r \|^2 \right) - \mathbf{R}_{lt} \mathbf{w}_t \left( \mathbf{w}_t^H \tilde{\mathbf{R}}_0 \mathbf{w}_t \right) - \lambda \mathbf{w}_t^H \mathbf{w}_t = 0 \quad (4.24)
\]

Applying the condition of \( \mathbf{w}_t^H \mathbf{w}_t = 1 \), (4.24) is rewritten as

\[
\frac{(\mathbf{w}_t^H \tilde{\mathbf{R}}_0 \mathbf{w}_t) \mathbf{w}_t^H \mathbf{R}_{lt} \mathbf{w}_t + \mathbf{w}_t^H \tilde{\mathbf{R}}_0 \mathbf{w}_t \| \mathbf{w}_r \|^2 - \mathbf{w}_t^H \mathbf{R}_{lt} \mathbf{w}_t (\mathbf{w}_t^H \tilde{\mathbf{R}}_0 \mathbf{w}_t)}{(\mathbf{w}_t^H \mathbf{R}_{lt} \mathbf{w}_t + \gamma \| \mathbf{w}_r \|^2)^2} - \lambda = 0 \quad (4.25)
\]

Therefore, the parameter \( \lambda \) is given by

\[
\lambda = \frac{\mathbf{w}_t^H \tilde{\mathbf{R}}_0 \mathbf{w}_t \gamma \| \mathbf{w}_r \|^2}{(\mathbf{w}_t^H \mathbf{R}_{lt} \mathbf{w}_t + \gamma \| \mathbf{w}_r \|^2)^2} \quad (4.26)
\]
4.2 Transmit and Receive Weight Vectors Determination

Substituting (4.26) and (4.20) into (4.23). We have

\[
\tilde{R}_0 w_t (w_t^H R_{lt} w_t + \gamma \| w_r \|^2) = w_t^H \tilde{R}_0 w_t (R_{lt} + \gamma \| w_r \|^2) w_t
\]

\[
(A^{(o)})^H w_r w_t^H (w_t^H R_{lt} w_t + \gamma \| w_r \|^2) = w_t^H (A^{(o)})^H w_r w_t^H (R_{lt} + \gamma \| w_r \|^2) w_t
\]

(4.27)

Since \( w_t^H A^{(o)} w_t \) is a scalar, (4.27) is rewritten as

\[
(A^{(o)})^H w_r = \frac{w_t^H (A^{(o)})^H w_r}{w_t^H R_{lt} w_t + \gamma \| w_r \|^2} (R_{lt} + \gamma \| w_r \|^2) w_t
\]

(4.28)

Let us define \( \xi \equiv \frac{w_t^H (A^{(o)})^H w_r}{w_t^H R_{lt} w_t + \gamma \| w_r \|^2} \), then the equation (4.28) can be expressed as

\[
w_t = \xi R_{nt}^{-1} (A^{(o)})^H w_r
\]

(4.29)

where

\[
R_{nt} = R_{lt} + \gamma \| w_r \|^2
\]

(4.30)

Without detailed calculation of \( \xi \), the optimal transmit weight vector can be given by

\[
w_t^{(opt)} = \frac{R_{nt}^{-1} (A^{(o)})^H w_r}{\| R_{nt}^{-1} (A^{(o)})^H w_r \|}
\]

(4.31)

We conclude that the determination of transmit and receive weight vectors by pointing out that the receive weight vector is estimated by (4.17) with the given transmit weight vector. Similarly for the transmit weight vector, it is estimated by (4.31) with the given receive weight vector. However, because the simultaneous equations become a nonlinear formula, it seems not easy to solve analytically. Therefore, an iterative algorithm, which estimates the receive and transmit weight vectors alternately is a simple way to reach solution for maximizing SINR. The iterative operation \((u > 1)\) is described in the next section.

4.2.3 Iterative Weight Vector Update Algorithm

For the updated receive weight vector

The updated receive weight vector at time \( u \) is given by

\[
\tilde{w}_r(u) = R_{nt}^{-1} (u - 1) A^{(o)} w_t(u - 1)
\]

(4.32)
4.3 Interference Cancellation Ability Analysis

where

\[ \mathbf{R}_{nr}(u) \equiv \sum_{l=1}^{L} \mathbf{A}^{(l)} \mathbf{w}_t(u) \mathbf{w}_t^H(u) \mathbf{A}^{(l)}^H + \gamma \mathbf{I} \]  

(4.33)

and

\[ \mathbf{w}_r(u) = \frac{\overline{\mathbf{w}}_r(u)}{\| \overline{\mathbf{w}}_r(u) \|} \]  

(4.34)

\( \mathbf{R}_{nr}(u - 1) \) is updated based on the previous transmitted weights \( \mathbf{w}_t(u) \) for finding the optimal receive weights \( \mathbf{w}_r(u) \).

**For the updated transmit weight vector**

The update transmit weight vector at time \( u \) is given by

\[ \mathbf{w}_t(u) = \mathbf{R}_{nt}^{-1}(u)(\mathbf{A}^{(o)})^H \mathbf{w}_r(u) \]  

(4.35)

where

\[ \mathbf{R}_{nt}(u) \equiv \sum_{l=1}^{L} (\mathbf{A}^{(l)})^H \mathbf{w}_r(u) \mathbf{w}_r^H(u) \mathbf{A}^{(l)} + \gamma \mathbf{I} \]  

(4.36)

and

\[ \mathbf{w}_t(u) = \frac{\overline{\mathbf{w}}_t(u)}{\| \overline{\mathbf{w}}_t(u) \|} \]  

(4.37)

Similarly, \( \mathbf{R}_{nt}(u) \) is also updated based on the previous received weights \( \mathbf{w}_r(u) \), for finding the optimal transmit weights \( \mathbf{w}_t(u) \).

In this work, the initial transmit weight vector \( \mathbf{w}_t(0) \) for \( u = 0 \) is determined from MIMO system in flat fading channel environment as follows.

\[ \mathbf{w}_t^{(opt)} \{ = \mathbf{w}_t(0) \} = \mathbf{e}_{t,max} \]  

(4.38)

The vector \( \mathbf{e}_{t,max} \) is the eigenvector corresponding to the largest eigenvalue \( \lambda_{max} \), for the correlation matrix \( (\mathbf{A}^{(o)})^H \mathbf{A}^{(o)} \) [43],[56].

4.3 Interference Cancellation Ability Analysis

Since the transmit and receive weight vectors are alternately updated based on the proposed iterative update weight algorithm, the DOFs of both the transmitter and receiver antennas are combined together to suppress the delayed versions of the preceding signal. Thus, for a MIMO system with \( M \) and \( N \) antennas equipped at
both the transmitter and the receiver, respectively, the maximum number of delayed
channels cancellation when jointing DOF of both sides is expected as

\[ L = (M - 1) + (N - 1) = M + N - 2 \] (4.39)

The maximum number of delayed channels cancellation is clearly confirmed by
computer simulation.

4.4 Output SINR

Based on the proposed method, the optimal transmit and receive weight vectors
are used in the transmitter and receiver, respectively for the single data stream
transmission. The output signal at the receiver side is given by

\[ y(t) = \sum_{l=0}^{L} (w_{r}^{opt})^{H} A^{(l)} (w_{t}^{opt}) s(t - l\Delta \tau) + (w_{r}^{opt})^{H} n(t) \] (4.40)

The output SINR of the proposed scheme is calculated via the cross-correlation
coefficient and given by

\[ \rho = \frac{E[y(t)s^{\ast}(t)]}{\sqrt{E[|y(t)|^{2}]E[|s(t)|^{2}]}} \] (4.41)

The output SINR is finally computed via the correlation coefficient as

\[ \text{SINR}_{\text{out}} = \frac{\rho^2}{1 - \rho^2} \] (4.42)

4.5 Simulation Results

4.5.1 Simulation Conditions

In this section, we offer the simulation conditions to demonstrate our analysis.
We assumed that the components of the channel state matrix are independent
identically distributed (i.i.d) with the uniform power delay profile (model 1) and
the exponential power delay profile (model 2) [55], respectively, given by

\[ p(\tau) = \sum_{l=0}^{L} \delta(\tau - lT_s) \quad \text{(model 1)} \] (4.43)
4.5 Simulation Results

\[ p(\tau) = \frac{P_R}{\sigma_\tau} \sum_{l=0}^{L} e^{-\frac{\tau}{\sigma_\tau}} \delta(\tau - lT_s) \] (model 2) \hfill (4.44)

where \( T_s \) is the symbol period of the transmitted signal, \( P_R \) denotes the average power of multipath signals, and \( \sigma_\tau \) represents the delay spread.

For model 1, each impulse \( a_{nm}^{(l)} \) is generated by the i.i.d. process while keeping \( E[|a_{nm}^{(l)}|^2] = 1 \) for \( n = 1 \sim N, m = 1 \sim M \) and \( l = 0 \sim L \). Figure 4.2(a) shows the assumed channel model considered in uniform power delay profile. Since the main purpose of this paper is to establish the weights determination scheme and identify the limit of delayed channels cancellation by means of our proposed scheme, we adopted an unrealistic propagation channel and a high SNR. In order to clarify the limit of the delayed channels cancellation ability, we set a sufficiently high \( P_s/P_N \), such that \( P_s/P_N = 40dB \).

Although the uniform power delay profile is effective for examination of the working mechanism of the proposed scheme, it is not realistic in indoor and outdoor wireless propagation environments. Therefore, model 2 will be used in order to evaluate the quantitative characteristics. Figure 4.2(b) shows the assumed channel model considered in the exponential power delay profile.

For both profiles, the amplitude of each impulse follows independent complex Gaussian distribution where the average follows (4.43) or (4.44). The simulation conditions are summarized in Table 4.1.

Table 4.1: Simulation model for MIMO beamforming

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of array</td>
<td>linear</td>
</tr>
<tr>
<td>((M, N)) antennas</td>
<td>(2,2), (3,3), (4,4), (8,8)</td>
</tr>
<tr>
<td>Type of modulation</td>
<td>BPSK</td>
</tr>
<tr>
<td>Signal length</td>
<td>(10^4) symbols</td>
</tr>
<tr>
<td>Sampling type</td>
<td>1 sample/symbol</td>
</tr>
<tr>
<td>Input SNR</td>
<td>20dB, 40dB</td>
</tr>
</tbody>
</table>
4.5 Simulation Results

4.5.2 Results

First, we carried out a $4 \times 4$ MIMO simulation for obtaining the output SINR comparison in the cases of flat and frequency-selective fading channel environments. The transmit and receive weight vectors of MIMO system in flat fading channel environment are estimated from finding the largest eigenvalues for the correlation matrices $(A^{(0)})^H A^{(0)}$ and $A^{(0)}(A^{(0)})^H$, respectively [56]. Over which those transmit and receive weight vectors are also used in transmission suffering the frequency-selective fading channels. The output SINR of the method is viewed in the channel with one symbol delay ($L = 1$). For comparison, we performed an iterative algorithm using one snapshot with iteration index $u$, running from 1 to 25. The SINR results are shown in Figure 4.3.

In Figure 4.3, due to the combining gain of the incoming multipath signals for the multipath flat fading channel, the output SINR curve increases slightly compared to that of the single path fading channel. It was shown that the output SINR curve is degraded severely in the frequency-selective fading channel by using the transmit and receive weight vector for MIMO flat fading channel. Obviously, the optimal transmit and receive weight vectors for MIMO flat fading channel cannot apply to MIMO frequency-selective fading channels.

Next, we performed a simulation for the $4 \times 4$ MIMO system over multipath frequency-selective channels using model 1 based on the iterative update algorithm of transmit and receive weight vectors, which described in the previous section. To show clearly the sufficient increase in the output SINR by using our proposed method for frequency-selective fading channels, we employ the weight vector estimated from the Singular Value Decomposition (SVD) method for MIMO flat fading channel as the initial weight vector. Figure 4.4 illustrates the averaged convergence characteristics of the system for different delays $L$ with 100 times under a random i.i.d. channel condition. It is seen that the maximum number of delayed channels cancellation of the $4 \times 4$ MIMO system is 6, which agrees very well with the result of $(M + N - 2)$, shown in equation (4.39). It should also be noted that $k = 80$ is a point having sufficient convergence for the $4 \times 4$ MIMO system undergoing 6 delayed channels.

Figure 4.5 shows the cumulative distribution function (CDF) of the output
4.5 Simulation Results

SINR for the $4 \times 4$ MIMO system after 100 random trials with the input SNR set to 40dB. The CDF yields the output SINR probability that the proposed MIMO cancels the number of delayed channels. As can be seen in Figure 4.5, the output SINR curves decrease with the increase in the number of delayed channels. However, the proposed MIMO effectively cancels the 6 delayed channels seen at 50 percent of the CDF.

The output SINR efficiency of the proposed $4 \times 4$ MIMO system is shown in Figure 4.6 for viewing the number of delayed channels in cases of $L = 6$ and $L = 7$ with the input SNR ranging from 0dB to 40dB. The output SINR for $L = 6$ increases linearly corresponding to the input SNR while the output SINR for $L = 7$ increases slightly. From these curves, we indeed to see that when the number of delayed channels is less than the limitation of number of delayed channels cancellation (or the joint DOF of transmit and receive antennas), the bit error rate of the received signal is smaller with the input SINR increasing from 0dB to 40dB. The bit error rate of the received signal is improved inconsiderably based on our proposed scheme when the number of delayed channels is less or equal to $(M + N - 2)$.

In order to confirm the maximum cancellation of delayed channels of the $M \times N$ MIMO system, we performed a simulation for the cases of $2 \times 2$, $3 \times 3$ and $4 \times 4$ MIMO systems in terms of the output SINRs at 50 percent of the CDF. The median value of SINRs versus the number of delayed channels are shown in Figure 4.7. It is clear that the proposed beamforming method can effectively cancel $(M + N - 2)$ delayed channels, namely, 2, 4 and 6 for the $2 \times 2$, $3 \times 3$ and $4 \times 4$ MIMO systems.

We are now going to show how our proposed method can apply to the frequency-selective fading channels by using model 2 with the delay spread of multipath waves over one symbol period. We performed a simulation for the cases of $1 \times 1$ (SISO), $2 \times 2$, $3 \times 3$, $4 \times 4$ and $8 \times 8$ MIMO systems with the delay spread $\sigma_{\tau}$ ranging from 1 to $10T_s$. The average SINR results are shown in Figure 4.8. It was shown that based on our proposed method, the increase in antenna elements at both ends improve considerably the output SINR when the delay spread of multipath waves is greater than one symbol period.

Successively, we performed a simulation for the cases of $2 \times 2$, $3 \times 3$, $4 \times 4$ and $8 \times 8$ MIMO systems in terms of the output SINR at 14dB, which is considered to be an acceptable condition in wireless communication systems. The transmit and
receive antennas of the $M \times N$ MIMO beamforming systems with ($M = N$) versus the delay spread $\sigma_{\tau}$ is shown in Figure 4.9. It was shown that $\sigma_{\tau} = (N - 1)T_s$ for ($M = N$) is a maximum delay spread where a reliable communication can be achieved by the proposed beamforming method.

4.6 Summary

In this paper, we proposed the weights determination scheme for MIMO beamforming in frequency-selective fading channels. This scheme yields the output SINR improvement based on the iterative update algorithm of the transmit and receive weight vectors and enables the cancellation of $(M + N - 2)$ delayed channels. The simulation result shows that the convergence speed of our proposed method is fast with knowledge of the CSI at both the transmitter and receiver.
Figure 4.2: Assumption of power delay profiles: a. A discrete-time uniform power delay profile; b. A discrete-time exponential power delay profile.
Figure 4.3: SINR in flat and frequency-selective fading channels when using $w_t$ and $w_r$ calculated from $A^{(0)}$.

Figure 4.4: Convergence characteristics for different delays.
Figure 4.5: CDF of SINR for the 4x4 MIMO at the receiver.

Figure 4.6: SINR performance of the proposed 4x4 MIMO system in case of $L = 6$ and $L = 7$. 
4.6 Summary

Figure 4.7: Median value of SINR vs. the number of delayed channels.

Figure 4.8: Average SINR of MxN MIMO as a function of the delay spread $\sigma_T$. (model 2)
Figure 4.9: The maximum delay spread realizing acceptable communication quality as a function of the number of array elements. (model 2)
Chapter 5

Multiuser MIMO Beamforming for Single Data Stream Transmission in FSF Channels

In this chapter, we propose a MIMO beamforming scheme using a single data stream transmission for multiuser systems in frequency-selective multipath channels. Assume that the channel state information (CSI) is perfectly known at both transmitter and receiver. The transmit weight vector for each user and the receive weight vector at base station (BS) corresponding to each user are calculated using two proposed algorithms, which maximize the output SINR for all receivers and every single link at the same time. Deriving MIMO beamforming for the single user system from Chapter 4, we analyze a maximum number of interference channels which could be eliminated in multiuser system.

5.1 Propagation Model

Suppose there are $Q$ users communicating with a BS. The $i$th user has a linear array antenna with $M_i$ elements, where $i = 1, \ldots, Q$. BS has an adaptive antenna with $N$ elements. A general MIMO beamforming for a single data stream transmission for multiuser system under frequency-selective fading channels environment is shown in Figure 5.1.

For wireless broadband systems, the MIMO propagation channel can be modeled as

$$H_i(\tau) = \sum_{l=0}^{L_i-1} A_i^{(l)} \delta(\tau - l\Delta\tau) \quad (5.1)$$
where $A_{i}^{(0)}$ is the channel information of the preceding wave of the $i$th user, which we regard as the desired channel in this work, while $A_{i}^{(l)}$ ($l = 1, ..., L_i - 1$), is the $l$th delayed channel information of the preceding channel of the $i$th user, which we consider as interference channels. The notation $a_{nm,i}^{(l)}$ means the $l$th interference path response between the $m$th transmit antenna of the $i$th user and $n$th receive antenna of the BS. $\Delta \tau$ is the unit delay time, which corresponds to symbol period $T_s$, of the modulated signal.

Since the received signal is assumed stationary. the output of array elements
is combined with complex weight vector to extract the desired signal. Thus the received signal for the \( i \)th user at the BS is expressed as

\[
y_i(t) = \sum_{j=1}^{Q} \sum_{l=0}^{L_i-1} w_r^{H} A_j^{(l)} w_{t,j} s_j(t - l\Delta \tau) + w_r^{H} n(t)
\]  

(5.3)

where \( s_j(t) \) is the transmit data of the \( j \)th user, and \( n = \begin{bmatrix} n_1, n_2, \ldots, n_N \end{bmatrix}^T \) is the additive white Gaussian noise (AWGN) vector. The beamforming transmit and receive weight vectors \( w_{t,i} \) and \( w_{r,i} \) for the \( i \)th user are defined as

\[
w_{t,i} = [w_{t1,i}, w_{t2,i}, \cdots, w_{tM,i}]^T
\]  

(5.4)

\[
w_{r,i} = [w_{r1,i}, w_{r2,i}, \cdots, w_{rN,i}]^T
\]  

(5.5)

Assume that the source signals are mutually uncorrelated with zero-mean and unit variance, thereby

\[
\langle s_i^*(t - l\Delta \tau)s_i(t - k\Delta \tau) \rangle = 0 \quad \text{for} \quad l \neq k
\]  

(5.6)

\[
\langle s_i^* (t)s_j(t) \rangle = 0 \quad \text{for} \quad i \neq j
\]  

(5.7)

Let us define \( P_{s,i}, P_N \) and \( 1/\gamma_i \), representing the signal power, noise power and power ratio of the signal to noise for the \( i \)th user:

\[
\langle |s_i|^2 \rangle = P_{s,i}
\]  

(5.8)

\[
\langle |n_1|^2 \rangle = \langle |n_2|^2 \rangle = \cdots = \langle |n_N|^2 \rangle = P_N
\]  

(5.9)

\[
1/\gamma_i \equiv P_{s,i}/P_N
\]  

(5.10)

The output SINR at the BS for the \( i \)th user is given by

\[
\Gamma(w_{t,i}, w_{r,i}) = \frac{w_r^{H} A_i^{(o)} w_{t,i} w_{t,i}^H (A_i^{(o)})^H w_{r,i}}{R_{inf} + \gamma_i w_r^{H} w_{r,i}}
\]  

(5.11)

where \( R_{inf} \) is the sum of interference channels of the preceding channels of the \( i \)th user and interference channels caused by other users.

\[
R_{inf} = \sum_{l=1}^{L_i-1} w_r^{H} A_i^{(l)} w_{t,i} w_{t,i}^H (A_i^{(l)})^H w_{r,i}
\]  

\[
+ \sum_{j=1,j\neq i}^{Q} \sum_{l=0}^{L_j-1} w_r^{H} A_j^{(l)} w_{t,j} w_{t,j}^H (A_j^{(l)})^H w_{r,i}
\]  

(5.12)
Similarly to Chapter 4, since the transmit and receive weight vectors for detection of the \(i\)th user are contained in numerator and denominator, (5.11) is a multivariable nonlinear equation. It seems to be difficult to find the optimal transmit and receive weight vectors analytically. Determination of the transmit and receive weight vectors for detection of the \(i\)th user under the effects of frequency-selective fading channels and multiuser is described in the next section.

5.2 Transmit and Receive Weight Vectors Determination

We employ the MSINR method to determine the optimal transmit and receive weight vectors, for which single user case is described in detail in [57]. By extending the method to multiuser case, the receive and transmit weight vectors determination for the \(i\)th user is summarized in the following.

5.2.1 Receive weight vector determination

Assume that all the transmit weight vectors \((w_{t,j}, j = 1, ..., Q)\) are given, and an optional condition of \(w_{r,i}^H w_{r,i} = 1\) to keep the total noise power constant. The steering vector of the desired signal is \(A_{i}^{(0)} w_{t,i}\) and based on the MSINR method, the receive weight vector at BS for the \(i\)th user is given by

\[
\begin{align*}
    w_{r,i}^{(opt)} &= R_{nr,i}^{-1} A_{i}^{(0)} w_{t,i} / \| R_{nr,i}^{-1} A_{i}^{(0)} w_{t,i} \| \\
    &= R_{nr,i}^{-1} A_{i}^{(0)} w_{t,i} / \| R_{nr,i}^{-1} A_{i}^{(0)} w_{t,i} \| \
\end{align*}
\]

where \(R_{nr,i}\) is a covariance matrix, which includes total information of interference channels of the \(i\)th user and interference channels caused by other users, given by

\[
R_{nr,i} = \sum_{l=1}^{L_i-1} A_{i}^{(l)} w_{t,i} w_{t,i}^H (A_{i}^{(l)})^H + \sum_{j=1,j\neq i}^{Q} \sum_{l=0}^{L_j-1} A_{j}^{(l)} w_{t,j} w_{t,j}^H (A_{j}^{(l)})^H + \gamma_i I
\]

5.2.2 Transmit weight vector determination

Assume that the receive weight vector \(w_{r,i}\) and all the transmit weight vectors excluding \(w_{t,i}\) are given, we determine the optimal transmit weight vector satisfying
the condition $w_{t,i}^H w_{t,i} = 1$. Based on the method of Lagrange multiplier, the optimal transmit weight vector for the $i$th user is given by

$$w_{t,i}^{(opt)} = R_{nt,i}^{-1} (A_i^{(o)})^H w_{r,i} / \|R_{nt,i}^{-1} (A_i^{(o)})^H w_{r,i}\|$$

(5.15)

where $R_{nt,i}$ is a covariance matrix, which includes total information of interference channels of the $i$th user and interference channels caused by other users, given by

$$R_{nt,i} \equiv \sum_{l=1}^{L_i-1} (A_i^{(l)})^H w_{r,i} w_{r,i}^H A_i^{(l)} + \sum_{j=1,j\neq i}^{Q} \sum_{l=0}^{L_j-1} w_{t,j}^H (A_j^{(l)})^H w_{r,i} w_{r,i}^H A_j^{(l)} w_{t,j} I + \gamma_i I$$

(5.16)

The detailed transmit weight vector determination is seen in the Appendix A.

We conclude that by giving the transmit weight vector of the $i$th user and fixing all the transmit weight vectors of the rest of users, the optimal receive weight vector at BS for the $i$th user is calculated by Eq.(5.13). Similarly, the optimal transmit weight vector for the $i$th user is calculated through Eq.(5.15) by giving the receive weight vector at the BS for the $i$th user and all the transmit weight vectors excluding $w_{t,i}$.

### 5.2.3 Iterative Weight Update Algorithms

In this section, we propose two algorithms along the lines of optimization principles. Block diagram of algorithm A and algorithm B are shown in Figure 5.2 and 5.3, respectively.

Given

| $w_{t,1}^{(0)}$ | $w_{t,2}^{(0)}$ |

Optimized

| $w_{r,1}^{(1)}$ | $w_{r,2}^{(1)}$ |

For what

| SINR of $y_1$ for $s_1$ | SINR of $y_2$ for $s_2$ |

Figure 5.2: Block diagram of the maximizing SINR by means of receiver-side weight vector optimization. ($Q = 2$) [Algorithm A]
For simplicity, it is assumed that there are two users communicating with a BS and selection of the initial weight vectors $w_t,1(0), w_t,2(0)$ for $k = 0$, where $k$ is iteration index for optimization, are determined from MIMO flat fading channel environments as follows.

$$w_{t,i}(0) = e_{t,i_{max}}$$  \hspace{1cm} (5.17)

The vector $e_{t,i_{max}}$ is the eigenvector corresponding to the largest eigenvalue $\lambda_{i_{max}}$ for the correlation matrix $(A_i^{(o)})^H A_i^{(o)}$ [56].

**Algorithm A: Maximizing SINR by receive weight vector optimization only**

Calculate the receive weight vectors $w_{r,1}(1), w_{r,2}(1)$ using the given transmit weight vectors $w_{t,1}(0), w_{t,2}(0)$ based on (5.13) to suppress interferences and maximize the output SINR corresponding to each user.

In this case, the transmit weight vector obtained from (5.17) is a premise to determine the optimal receive weight vector for maximizing the output SINR. Thus the only degree of freedom (DOF) of BS, namely $N - 1$, is consumed for cancellation of interference waves in desired signal and all other multipath waves in interfering signals. Although only the DOF of BS is consumed, the algorithm itself is very
simple and seems practical.

**Algorithm B:** Maximizing SINR based on iterative update of both ends weight vectors

1. Calculate the receive weight vectors $w_{r,1}(k+1), w_{r,2}(k+1)$ with the given transmit weight vectors $w_{t,1}(k), w_{t,2}(k)$ based on (5.13) for maximizing SINR corresponding to each user.

2. Calculate the transmit weight vectors $w_{t,1}(k+1), w_{t,2}(k+1)$ by the updated receive weight and previous transmitted weight vectors $w_{r,1}(k+1), w_{t,2}(k)$ and $w_{r,2}(k+1), w_{t,1}(k+1)$, respectively, based on (5.15) for maximizing SINR corresponding to each user.

3. Go to step 1 and repeat the iteration until a certain condition for termination is satisfied.

4. Send the optimal transmit weight vector obtained at BS to mobile stations. Using this transmit weight vector at each user side, the SINR of each user is maximized at the BS.

The maximum SINR is given in (5.11) where $w_{t,i}$ and $w_{r,i}$ are replaced by $w_{t,i}^{(opt)}$ and $w_{r,i}^{(opt)}$, respectively. Although both the algorithm A and B maximize SINR, the algorithm A is a scheme which optimizes the weight vector of receiver side only while the algorithm B is an algorithm which optimizes the weight vector of both transmitter and receiver at BS. Thus, condition is different even if it is the same propagation environment, the performance (namely, obtained maximum SINR) of the algorithm B is better. The detailed analysis of this algorithm is carried out in the next section from the view point of DOF.

### 5.3 Interference Cancellation Ability Analysis

In this section, we investigate the interference channels cancellation ability of the proposed MIMO beamforming method in the single data stream transmission for multiuser system. In general, an $M$ element array, where the optimal beamformer...
5.3 Interference Cancellation Ability Analysis

Figure 5.4: Examples of the channel impulse response.

without using the tapped delay lines, has effectively cancelled $M-1$ independent waves, namely, interference waves [18],[22]. It seems correct for the Single-Input Multiple-Output (SIMO) or Multiple-Input Single-Output (MISO) system but the MIMO system. Actually, the interference channels cancellation of MIMO system has been larger than both SIMO and MISO systems, $(M-1)+(N-1)$ is maximum number of interference channels cancellation ability, which is proved from $M \times N$ MIMO system with respect to a single user communicating with the BS [57]. However, for multiuser system, not only interference channel of the preceding channel from the desired user, but also interference channels from other users should be cancelled. For instance, channel model for two users is shown in Figure 5.4, where the preceding channel impulses of the first user is regarded as the desired preceding path and all other multipath waves are interferences, which should be suppressed.

Let’s consider that the number of antennas for all users is 1 each. Since BS with $N$ antennas is able to cancel $N-1$ undesired users, the BS can accommodate $N$ users at most ($Q \leq N$). Next, the case where the number of each user is more than two transmit antennas is considered. Even in this case since the DOF of user’s antennas is used to control the weight vector for maximizing its SINR, it is not used to remove interference channels of the desired user. Therefore, condition ($Q \leq N$) is still satisfied for MIMO systems with our proposed scheme. However, when the DOF of transmitter antennas is utilized, not only the receive weight vector is updated in order to exploit the DOF of BS to cancel the undesired users but also the transmit weight vector is updated to produce the DOF of desired user for canceling
5.3 Interference Cancellation Ability Analysis

its own interference channels. They are alternately updated to eliminate much more interference channels. We summarize now the conditions to find a total number of interference channels cancellation as follows.

- A maximum number of users $Q$ is less than or equal to BS array antenna $N$ ($Q \leq N$).
- The DOF of BS, $N - 1$, could be used to cancel interference channels of any users.
- If the DOF of BS is larger than a total interference channels of all the undesired users, the DOF remainder of BS might be useful for the desired user to cancel more interference channels themselves.
- The DOF of the desired $i$th user, $M_i - 1$, could be used to cancel its own interference channels $\{A_i^{(l)}; l = 1, ..., L_i - 1\}$.
- The DOF remainder of each user cannot cancel interference channels of other users. In other words, BS can only utilize its own DOF remainder to cancel interference channels of users.

For the algorithm A, the condition for interferences cancellation for the $i$th user in multiuser system is found through an inequality, which set up based on the above conditions and is given by

$$N - 1 \geq L_U + (L_i - 1) \quad \text{for algorithm A} \quad (5.18)$$

where $L_U$ is a total number of interference channels of all the undesired users, and is given by

$$L_U = \sum_{j=1, j \neq i}^{Q} L_j \quad (5.19)$$

Based on the update of transmit and receive weight vectors, the BS exploits its DOF to cancel interference channels of all the undesired users and the DOF of desired user is used to cancel interference channels themselves. Therefore, a total number of interference channels cancellation for the $i$th user in multiuser system using the algorithm B is found through an inequality in the following.

$$N - 1 \geq L_U + L_D \quad \text{for algorithm B} \quad (5.20)$$
where $L_D$ is the remainder of interference channels of the desired user after using its DOF to cancel interference channels themselves, and is given by.

$$L_D = \max\{(L_i - 1) - (M_i - 1), 0\}$$  \hspace{1cm} (5.21)

In the following, the effectiveness of above inequalities (5.18) and (5.20) are clearly confirmed by the computer simulation.

### 5.4 Output SINR

Based on the proposed method for multiuser system, the optimal transmit and receive weight vectors are used in the transmitters and receiver, respectively for single data stream transmission. The output signal of the $i$th user at the BS is given by

$$y_i(t) = \sum_{j=1}^{Q} \sum_{l=0}^{L_j-1} (w_{r,i}^{opt})^H A_j^{(l)} w_{t,j}^{opt} s_j(t - l\Delta\tau) + (w_{r,i}^{opt})^H n(t)$$  \hspace{1cm} (5.22)

where $w_{t,j}^{opt}$ and $w_{r,i}^{opt}$ are the optimal transmit and receive weight vectors, respectively, obtained from either the algorithm A or the algorithm B.

Then the output SINR of the $i$th user is calculated via the cross-correlation coefficient $\rho_i$ at the BS and given by

$$\rho_i = \frac{E[y_i(t) s_i^*(t) ]}{\sqrt{E[|y_i(t)|^2]E[|s_i(t)|^2]}}$$  \hspace{1cm} (5.23)

The output SINR calculation via the correlation coefficient for the $i$th user at the BS is finally given by

$$\text{SINR}_{out,i} = \frac{|\rho_i|^2}{1 - |\rho_i|^2}$$  \hspace{1cm} (5.24)

### 5.5 Simulation Results

#### 5.5.1 Simulation Conditions

In this section, to demonstrate the performance of the proposed algorithms, computer simulations are carried out for two users communicating with a BS case. We
assume that the components of the channel state matrix are independent identically
distributed (i.i.d) with complex Gaussian distribution, with the uniform power delay
profile (model 1) and the exponential power delay profile (model 2) \[55\], respectively,
given by

\[ p_i(\tau) = \sum_{l=0}^{L_i-1} \delta(\tau - lT_s) \quad \text{(model 1)} \]  
\[ p_i(\tau) = \frac{P_{R,i}}{\sigma_{\tau,i}} \sum_{l=0}^{L_i-1} e^{-\frac{\tau}{\sigma_{\tau,i}}} \delta(\tau - lT_s) \quad \text{(model 2)} \]  

where \( T_s \) is the symbol period of the transmitted signal, and \( P_{R,i} \) is the average
power of multipath waves and \( \sigma_{\tau,i} \) is the delay spread of the \( i \)th user channel.

For model 1, each impulse \( a_{nm,i}^{(l)} \) is generated by the i.i.d. process while keeping
\( \langle |a_{nm,i}^{(l)}|^2 \rangle = 1 \) for \( n = 1 \sim N, m = 1 \sim M_i \) and \( l = 0 \sim L_i - 1 \). In order to clarify
the limit of the delayed channels cancellation ability, we set a sufficiently high
input SNR = 40dB. For model 2, the exponential power delay profile will be used
in order to have a more realistic channel model analyzing the performance of our
proposed method. To identify the improvement of the output SINR with respect to
the delay spread of multipath waves, we set the input SNR = 20dB. The simulation
conditions are summarized in Table 5.1.

Table 5.1: Simulation model for multiuser MIMO beamforming

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of array</td>
<td>linear</td>
</tr>
<tr>
<td>((M, N)) antennas</td>
<td>(2,2), (4,4), (4,6), (6,4), (6,6)</td>
</tr>
<tr>
<td>Type of modulation</td>
<td>BPSK</td>
</tr>
<tr>
<td>Signal length</td>
<td>(10^4) symbols</td>
</tr>
<tr>
<td>Sampling type</td>
<td>1 sample/symbol</td>
</tr>
<tr>
<td>Input SNR</td>
<td>20dB, 40dB</td>
</tr>
</tbody>
</table>

5.5.2 Results

First, we performed two users communicating with a BS simulation in multipath
frequency-selective fading channels environment using model 1 that shows the con-
vergence characteristics of system over interference channels by using the algorithm
B. The number of antenna elements of BS and each user are equal to 4. In order to make clear interference channels cancellation limit, the input SNR is set to be of 40dB. The reference source signal of each user is used in BS to detect the desired signal. The simulation result is shown in Figure 5.5.

Although the propagation environment of user 1 and 2 has the same statistical characteristic, instantaneous environment given by complex random number is different. Then the transmit weight vector of user 1 and 2 is optimized at BS with respect to its received signal. Thus, since the statistical characteristic of propagation environment and antennas number of two users are the same, two convergence characteristics are similar. After 100 times trials by changing the propagation conditions keeping the given statistical property constant, it can be identified that the system cancels a total number of 5 interference channels (namely, \( L_1 = L_2 = 3 \)) for both user 1 and user 2. Since the increase in interferences leads to the matrix be full rank so that the increase in interference channels lead to longer convergence time. It is seen that \( k = 80 \) is a point having sufficient convergence characteristic of system corresponding to 5 interference channels.

Next, for comparison of two proposed algorithms, we carried out a two-user 4×4 MIMO system simulation using model 1 that yields the cumulative distribution function (CDF) of SINR. The simulation of CDF shows that the proposed multiuser MIMO beamforming system cancels a number of interference channels effectively. The simulation result is shown in Figure 5.6.

As can be seen from the CDF for user 1 in Figure 5.6(a), user 1 and user 2 are considered to be the desired and undesired users, respectively. Let’s consider the case of \( (L_1 = L_2 = 2) \) with the transmit and receive antennas \( (M_1 = M_2 = N = 4) \), for instance. By substituting those values into inequalities (18) and (20), we have \( (3 \geq L_U + 1) \) for the algorithm A and \( (3 \geq L_U + 0) \) for the algorithm B. As a result, \( L_U = 2, 3 \) for the algorithm A and B, respectively, which verified by the result of simulation. Moreover, the \( L_U \) obtained from inequalities (18) and (20) also imply that the algorithm A cannot remove for \( (L_1 = 2, L_2 \geq 3) \) while the algorithm B cancels effectively for \( (L_1 = 4, L_2 = 3) \). The performance characteristics obtained by using the algorithm B is better than algorithm A since DOF of the desired user is effectively utilized to cancel many copies of its preceding channel.

Similarly, Figure 5.7 compares two proposed algorithms for a two-user MIMO
5.5 Simulation Results

system using model 1 where the number of antenna elements of each user is equal to 6 while the number of antenna elements of BS is 4.

As can be seen in Figure 5.7, when user 1 and user 2 are considered as the desired and undesired users, respectively, the system has an ability to cancel less than or equal 5 waves of 7 total interference channels using the algorithm B. On the contrary, when user 2 and user 1 are considered as the desired and undesired users, respectively, the system has an ability to cancel effectively the total interference channels. Similarly to system using the algorithm A, the simulation results obtained from the algorithm A and B agree very well with the relation given by inequalities (5.18) and (5.20). The algorithm B eliminates considerably the number of interference channels compared to the algorithm A.

Successively, we performed several two-user MIMO system simulations using model 1 to verify the relation of inequalities (5.18) and (5.20), which given in Table 5.2. Herein, $M_1, L_1$ and $M_2, L_2$ are the number of antennas and i.i.d. channels of user 1 and user 2. $N$ is the number of antennas of BS. O and X denote the possibility of interference channels cancellation satisfied and unsatisfied with inequalities (5.18) and (5.20), by judging whether the median value of SINR is larger than 30dB (O) or not (X). The computer simulations for the algorithm A and algorithm B are in good agreement with the results estimated from relation of inequalities (5.18) and (5.20), respectively. It is also observed that the algorithm B improves the performance of system when the preceding channel of the desired user has a lot of its own interference channels.

Figure 5.8 shows the SINRs viewing at 50 percent of the CDF versus the total number of interference and delayed channels of the $4 \times 4$, $4 \times 6$, and $6 \times 4$ MIMO systems. As can be seen in Figure 5.8, the total number of interference channels cancelled by exploiting the DOF of both the BS and users of the algorithm B is larger than that of the algorithm A. Since the DOF of the transmit antennas for each user is used to cancel its own interference channels, it is not available to eliminate interference channels of any other users. However, the DOF of the receive antennas for BS has capable to cancel interference channels for all users if the number of interference channels are within the DOF. Therefore, the increasing the number of antennas of BS is better than that of mobile stations to suppress interference channels.
5.6 Summary

We are now going to show how our proposed schemes apply to multiuser MIMO system in frequency-selective fading channels using model 2 for the delay spread of multipath waves over one symbol period. We performed a simulation for two-user in the cases of $4 \times 4$, $4 \times 6$, and $6 \times 4$ MIMO systems with the input SNR = 20dB and delay spread $\sigma_r$ ranging from 0 to $5T_s$. The simulation result is shown in Figure 5.9. It is shown that the performance of system obtained by using the algorithm B is better than the algorithm A and the increase in number of antennas at BS improves the SINR compared to the increase in number of antennas at the desired user.

5.6 Summary

We have proposed multiuser MIMO beamforming system for the single data stream transmission. The proposed algorithms are applied to multi-user systems in frequency-selective fading channels without using the tapped delay line structure. The algorithm A, which is characterized by simple scheme, showed good performance for propagation channel when the preceding channel of the desired user has few its own interference channels, while the algorithm B, which is more sophisticated scheme, shown better performance for canceling not only interference channels caused by other users but also more its own interference channels as shown in Figure 5.9. The improvement of SINR by the proposed algorithms allows that either the more interference channels can be suppressed or more users can be supported in multiuser MIMO systems where a maximum cancellation number of interference channels must satisfy with (5.18) and (5.20).
Figure 5.5: Convergence characteristics of $4 \times 4$ MIMO system for two users using model 1. (upper: for user 1, lower: for user 2)
Figure 5.6: Distribution function of SINR for two users 4 × 4 MIMO system: a. CDF for User 1 (upper); b. CDF for user 2 (lower).
Figure 5.7: Distribution function of SINR for two users $6 \times 4$ MIMO system.
Table 5.2: Interference channels cancellation ability of i.i.d channels of user 1 and user 2, respectively

<table>
<thead>
<tr>
<th>$M_1$</th>
<th>$M_2$</th>
<th>$N$</th>
<th>$L_1$</th>
<th>$L_2$</th>
<th>Algorithm</th>
<th>Desired User</th>
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<tbody>
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<td>B</td>
<td>O</td>
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</tbody>
</table>

O: Total interference channels cancellation is satisfied with inequalities (5.18), (5.20) or confirmed by simulation result viewing the SINR at 40dB and 50% of CDF.
X: Total interference channels cancellation is not satisfied with inequalities (5.18), (5.20) or verified by simulation result viewing the SINR at 40dB and 50% of CDF.
5.6 Summary

The number of interference channels:

\[ L_1( = L_2 ) \]

Figure 5.8: Median value of SINR as a function of \( L_1 \) in case of \( L_1 = L_2 \).

The number of interference channels: \( L_1 = L_2 \)

Figure 5.9: Average SINR of MIMO system as a function of the delay spread \( \sigma_r \).
Chapter 6

Spatial-Temporal MIMO Beamforming for Single Data Stream Transmission in FSF Channels

This chapter presents spatial-temporal MIMO beamforming for single carrier transmission in frequency-selective fading (FSF) channels with an assumption of perfect channel state information (CSI) at both sides. The transmit and receive weight vectors for detection of the preceding signal and the receive weight vectors for detection of the delayed versions of the preceding signal incoming BS at different time are estimated by using an iterative update algorithm. Based on minimum mean square error (MMSE) method, multiple delayed waves are exploited to maximize the output SINR instead of suppressing them at the receiver side. This improvement of output SINR is useful for MIMO beamforming systems to complement the high-quality in broadband wireless communications when the distance of transmission is made sufficiently large.

6.1 Propagation Model

Similarly to MIMO beamforming for single data stream transmission in frequency-selective fading channels, which has been proposed in Chapter 4, the MIMO propagation channel, where the number of transmitter and receiver antennas is $M$ and $N$, respectively, can be modeled as

$$H(\tau) = \sum_{l=0}^{L} A^{(l)} \delta(\tau - l\Delta\tau)$$

(6.1)
6.1 Propagation Model

\[
A^{(l)} = \begin{pmatrix}
  a_{11}^{(l)} & a_{12}^{(l)} & \cdots & a_{1M}^{(l)} \\
  a_{21}^{(l)} & a_{22}^{(l)} & \cdots & a_{2M}^{(l)} \\
  \vdots & \vdots & \ddots & \vdots \\
  a_{N1}^{(l)} & a_{N2}^{(l)} & \cdots & a_{NM}^{(l)}
\end{pmatrix}
\]  

(6.2)

where \( A^{(0)}(l = 0) \) is the channel information of the preceding wave which we regard as the desired channel in this work. \( A^{(l)}(l = 1, \ldots, L) \) is the \( l \)th delayed channel information. The notation \( a_{nm}^{(l)} \) means the \( l \)th delayed path gain response between the \( m \)th transmit antenna and \( n \)th receive antenna. \( \Delta \tau \) is the unit delay time, which corresponds to symbol period \( T_s \), of the modulated signal.

### 6.1.1 Spatial-domain Signal Processing

A spatial-domain MIMO beamforming for a single data stream transmission, which has proposed in [57], under FSF channels is shown in Figure 6.1.

The output of array elements is linearly combined with weight vector to give the received signal. The output signal at the receiver can be expressed as

\[
y(t) = \sum_{l=0}^{L} w_r^H A^{(l)} w_t s(t - l\Delta \tau) + w_r^H n(t)
\]

(6.3)

where \( s(t) \) is the transmitted signal, and \( n = [n_1, n_2, \ldots, n_N]^T \) is i.i.d. additive noise vector, which is assumed to be complex Gaussian processes with zero-mean and
variance $N_0/2$. The beamforming transmit and receive weight vectors $w_t, w_r$ are defined as

$$w_t = [w_{t1}, w_{t2}, \cdots, w_{tM}]^T \quad (6.4)$$

$$w_r = [w_{r1}, w_{r2}, \cdots, w_{rN}]^T \quad (6.5)$$

Assume that each delayed signal is uncorrelated and zero-mean, thereby

$$\langle s^*(t - i\Delta\tau)s(t - j\Delta\tau) \rangle = 0 \quad \text{for} \quad i \neq j \quad (6.6)$$

Let us define $P_s, P_N$ and $1/\gamma$ representing the signal power, noise power and power ratio of the signal to noise.

$$\langle |s|^2 \rangle = P_s \quad (6.7)$$

$$\langle |n_1|^2 \rangle = \langle |n_2|^2 \rangle = \cdots = \langle |n_N|^2 \rangle = P_N \quad (6.8)$$

$$1/\gamma \equiv P_s/P_N \quad (6.9)$$

If CSI of the preceding channel $A^{(0)}$ is known well at both sides, the output SINR at the receiver is given by

$$\Gamma(w_t, w_r) = \frac{w^H_t A^{(0)} w_r (A^{(0)})^H w_r}{\sum_{l=1}^{L} w^H_{rl} A^{(l)} w_t (A^{(l)})^H w_r + \gamma w^H_r w_c} \quad (6.10)$$

Since both the transmit and receive weight vectors are contained in the numerator and denominator, (6.10) is considered a multivariable nonlinear equation. It seems to be difficult to find the optimal weight vectors for the transmitter and receiver analytically for maximizing the output SINR at the receiver. The solution to find the optimal receiver and transmitter weight vectors for MIMO system in FSF channels for detecting only the preceding signal is based on an iterative weight update algorithm, which has been proposed in the Chapter 4. In the proposed algorithm, the receiver and transmitter weight vectors are calculated and updated alternately by using MMSE criterion and Lagrange method, respectively. Briefly, the iterative operation $(u > 1)$ is described as follows.

For the receiver weight vector

$$w_r(u) = \frac{w_r(u)}{\|w_r(u)\|} \quad (6.11)$$
where
\[ w_r(u) = R_{nr}^{-1}(u-1)A^{(o)}w_t(u-1) \quad (6.12) \]
and
\[ R_{nr}(u) \equiv \sum_{l=1}^{L} A^{(l)}w_t(u)w_r^H(u)(A^{(l)})^H + \gamma I \quad (6.13) \]

\[ R_{nr}(u-1) \] is updated based on the previous transmitted weight vector \( w_t(u) \) for finding the optimal receive weight vector \( w_r(u) \).

**For the transmitter weight vector**
\[ w_t(u) = \bar{w}_t(u)/\|\bar{w}_t(u)\| \quad (6.14) \]
where
\[ w_t(u) = R_{nt}^{-1}(u)(A^{(o)})^Hw_r(u) \quad (6.15) \]
and
\[ R_{nt}(u) \equiv \sum_{l=1}^{L} (A^{(l)})^Hw_r(u)w_r^H(u)A^{(l)} + \gamma I \quad (6.16) \]

Similarly, \( R_{nt}(u) \) is also updated based on the previous received weight vector \( w_r(u) \) for finding the optimal transmit weight vector \( w_t(u) \).

Based on the proposed scheme, the degree of freedom (DOF) of both the transmitter and receiver are joined together to suppress the delayed versions of the preceding signal while maximizing the output SINR at the receiver. The maximum number of the delayed channels cancellation ability for \( M \times N \) MIMO system is confirmed by the computer simulation and given by
\[ L = M + N - 2 \quad (6.17) \]

Deriving from (6.17), the transmitter and receiver contributed their maximum DOF through out the proposed method to mitigate effectively the delayed versions of the preceding signal without using the tapped delayed line structure.

### 6.1.2 Spatial-Temporal Signal Processing

Although the spatial-domain adaptive MIMO beamforming had proposed to suppress ISI, since the preceding signal is considered as the desired signal and the
subsequent signals are referred to as interferences reflected at increasing distance of transmission, the more delayed signals increase the more output SINR at receiver is degraded. In this paper, we propose spatial-temporal MIMO beamforming that improve the output SINR at receiver by utilizing multiple delayed versions of the preceding signal instead of suppressing them. The spatial-temporal MIMO beamforming configuration is shown in Figure 6.2.

![Spatial-Temporal MIMO Beamforming configuration for FSF Channels.](image)

Figure 6.2: Spatial-Temporal MIMO Beamforming configuration for FSF Channels.

Assume that the delayed channels $A^{(k)}$ for $k = 1, \ldots, K < L$ are known as well as the preceding channel $A^{(0)}$, the delayed versions of the transmitted signal are uncorrelated and might be also detected simultaneously at the receiver.

By multiplying the receive weight vector $w_r^{(k)}$, the output signal arrived at time $k$ at the receiver can be expressed as

$$\hat{s}^{(k)}(t) = w_r^{(k)}A^{(k)}w_t^{(0)}s(t - k\Delta\tau) + \sum_{l=0,l\neq k}^L w_r^{(k)}A^{(l)}w_t^{(0)}s(t - l\Delta\tau) + w_r^{(k)}n(t)$$

(6.18)

Since the path lengths of the preceding and delayed signals are different, the received signal $\hat{s}^{(k)}(t)$ has the same data as the preceding signal $\hat{s}^{(0)}(t)$ but it is
shifted by amount of \( k \) symbols delay time. In order to utilize the detected signals \( \hat{s}^{(0)}(t) \) to \( \hat{s}^{(K)}(t) \) for maximizing the output SINR at the receiver, they are led into a temporal-domain system, where the received signal \( \hat{s}^{(k)}(t) \) is retarded by \( (K - k) \) symbols time to form all the received signals having the same amount of delay time. Then, the output array elements are combined with the optimal weight vector to recover the data with highest output SINR. The output signal after spatial-temporal signal processing can be given by

\[
y(t) = \sum_{k=0}^{K} (w_c^{(k)})^* \hat{s}^{(k)}(t) = w_c^H \hat{s}(t) \tag{6.19}
\]

If the optimal transmit and receive weight vectors are given, the overall SINR at the receiver based on the spatial-temporal adaptive signal processing is expected by

\[
\text{SINR}_{\text{out}} = \text{SINR}_0 + \sum_{k=1}^{K} \text{SINR}_k \tag{6.20}
\]

where \( \text{SINR}_0 \) and \( \text{SINR}_k \) is the output SINR obtained from the preceding signal \( \hat{s}^{(0)}(t) \) and the delayed signal \( \hat{s}^{(k)}(t) \), respectively.

### 6.2 Weights Determination of the Proposed Spatial-Temporal Adaptive MIMO Beamforming

The optimal transmit and receive weight vectors of the spatial-temporal signal processing, which combines the delayed signals with the preceding signal for maximizing the output SINR at the receiver, are illustrated as follows.

- Calculate the optimal receive and transmit weight vectors \( w_r^{(0)}(u) \), \( w_t^{(0)}(u) \) at the \( u \)th iteration for detection of the preceding signal, which has been proposed in [57], and given by

\[
w_r^{(0)}(u) = \frac{R_0^{-1}(u-1)A^{(0)}w_t^{(0)}(u)}{||R_0^{-1}(u-1)A^{(0)}w_t^{(0)}(u)||} \tag{6.21}
\]

\[
w_t^{(0)}(u) = \frac{R_0^{-1}(u)(A^{(0)})^H w_r^{(0)}(u)}{||R_0^{-1}(u)(A^{(0)})^H w_r^{(0)}(u)||} \tag{6.22}
\]
where
\[
\mathbf{R}_{r0}(u) = \sum_{l=1}^{L} \mathbf{A}^{(l)} \mathbf{w}_r^{(0)}(u)(\mathbf{w}_r^{(0)})^H(u)(\mathbf{A}^{(l)})^H + \gamma \mathbf{I} \quad (6.23)
\]
and
\[
\mathbf{R}_{t0}(u) = \sum_{l=1}^{L} (\mathbf{A}^{(l)})^H \mathbf{w}_r^{(0)}(u)(\mathbf{w}_r^{(0)})^H(u) \mathbf{A}^{(l)} + \gamma \mathbf{I} \quad (6.24)
\]

- Go to the \((u+1)\)th iteration until a certain condition for termination is satisfied.

\[
\mathbf{w}_r^{(0)} \rightarrow \mathbf{w}_r^{(0),opt} \quad (6.25)
\]

\[
\mathbf{w}_t^{(0)} \rightarrow \mathbf{w}_t^{(0),opt} \quad (6.26)
\]

- Calculate the optimal receive weight vector \(\mathbf{w}_r^{(k)}\) for detection of the \(k\)th delayed signal based on the optimal transmit weight vector \(\mathbf{w}_t^{(0),opt}\), which updated from the iterative algorithm.

\[
\mathbf{w}_r^{(k)} = \frac{\mathbf{R}_{rk}^{-1} \mathbf{A}^{(k)} \mathbf{w}_t^{(0),opt}}{||\mathbf{R}_{rk}^{-1} \mathbf{A}^{(k)} \mathbf{w}_t^{(0),opt}||} \quad (6.27)
\]

where
\[
\mathbf{R}_{rk} = \sum_{l=0,l \neq k}^{L} \mathbf{A}^{(l)} \mathbf{w}_t^{(0),opt}(\mathbf{w}_t^{(0),opt})^H(\mathbf{A}^{(l)})^H + \gamma \mathbf{I} \quad (6.28)
\]

- Calculate the weight vector \(\mathbf{w}_c^{opt}\) to maximize the output SINR at the receiver based on MMSE criterion.

\[
\mathbf{w}_c^{opt} = \arg \min_{\mathbf{w}_c} E\{|s(t) - \mathbf{w}_c^H \hat{s}(t)|^2\} \quad (6.29)
\]

where \(E\{\cdot\}\) represents the expected operation, and \(\hat{s}(t) = [\hat{s}^{(0)}(t) \ldots \hat{s}^{(K)}(t)]\). Taking the derivative of the argument of (6.29) with respective to weight vector \(\mathbf{w}_c\) then equating it to zero yields

\[
\mathbf{w}_c^{opt} = \mathbf{R}_{\hat{s}\hat{s}}^{-1} \mathbf{p}_{\hat{s}s} \quad (6.30)
\]

where
\[
\mathbf{R}_{\hat{s}\hat{s}} = E\{\hat{s}(t)\hat{s}^H(t)\} \quad (6.31)
\]

is the covariance matrix of the received signals and
\[
\mathbf{p}_{\hat{s}s} = E\{\hat{s}(t)s^*(t)\} \quad (6.32)
\]

is the correlation vector of the received signal \(\hat{s}(t)\) with reference data \(s(t)\).
Since the preceding channel $A^{(0)}$ is considered as the desired channel, the optimal transmit and receive weight vectors $w_{t}^{(0),opt}$ and $w_{r}^{(0),opt}$ are estimated based on the iterative update algorithm, which has been proposed in [57]. On the other hand, assume that the delayed channels $A^{(1)}, ..., A^{(K)} (K \leq L)$ are also known as well as the desired channel, the delayed versions of the preceding signal can be detected at the receiver side. Since the weight vector of the transmitter is updated to join the DOF of both the transmitter and receiver to suppress $(M + N - 2)$ interferences while maximizing the output SINR of the preceding signal, it does not provide its DOF for detection of the delayed signals. In other word, only DOF of the receiver is available to suppress $(N - 1)$ interferences for detection of the delayed signals.

### 6.3 Output SINR

Based on the proposed spatial-domain signal processing, the optimal transmit and receive weight vectors are used in the transmitter and receiver, respectively for single data stream transmission. The output signal of the $k$th received signal ($k = 0, 1, ..., K$) at the receiver side is given by

$$
\hat{s}^{(k)}(t) = w_{r}^{(k),opt} A^{(k)} w_{t}^{(0),opt} s(t - k\Delta \tau) \\
+ \sum_{l=0, l\neq k}^{L} w_{r}^{(k),opt} A^{(l)} w_{t}^{(0),opt} s(t - l\Delta \tau) + w_{r}^{(k),opt} n(t)
$$

(6.33)

The output SINR of the $k$th received signal is calculated via the cross-correlation coefficient $\rho_k$ at BS and given by

$$
\rho_k = \frac{E[\hat{s}^{(k)}(t)s^*(t - k + 1)]}{\sqrt{E[|\hat{s}^{(k)}(t)|^2]E[|s(t - k + 1)|^2]}}
$$

(6.34)

The output SINR calculation via the correlation coefficient for the $k$th received signal at the BS is finally given by

$$
\text{SINR}_k = \frac{|\rho_k|^2}{1 - |\rho_k|^2}
$$

(6.35)

Based on temporal-domain signal processing, the output signal that combined the preceding signal and the delayed signals by multiplied the optimal weight vector
6.4 Simulation Results

is given by

\[ y(t) = \sum_{k=0}^{K} \left( w_c^{(k),opt} \right)^* s^{(k)}(t) \]  
(6.36)

The output SINR of spatial-temporal adaptive signal processing is calculated via the cross-correlation coefficient \( \rho \) at BS and given by

\[ \rho_k = \frac{E[y(t)s^*(t)]}{\sqrt{E[|y(t)|^2]E[|s(t)|^2]}} \]  
(6.37)

Finally, the output SINR calculation via the correlation coefficient is given by

\[ \text{SINR}_{\text{out}} = \frac{|\rho|^2}{1 - |\rho|^2} \]  
(6.38)

6.4 Simulation Results

6.4.1 Simulation Conditions

In this section, we carry out the performance analysis of the proposed spatial-temporal MIMO beamforming in frequency-selective fading channels using simulation results by computer programs. The computer simulations are performed for single data stream transmission in a single user case, where the MS is equipped with \( M = 4 \) antenna elements while the BS has \( N = \{4, 8\} \) antenna elements, which respectively results in \( 4 \times 4 \) and \( 4 \times 8 \) MIMO systems. We assume that the components of the channel state matrix are independent identically distributed (i.i.d) with complex Gaussian distribution, with the uniform power delay profile (model 1) and the exponential power delay profile (model 2) [55], respectively, given by

\[ p(\tau) = \delta(\tau - lT_s) \quad \text{(model 1)} \]  
(6.39)

\[ p(\tau) = \frac{P_R}{\sigma_T} \sum_{l=0}^{L} e^{-\frac{\tau}{\sigma_T}} \delta(\tau - lT_s) \quad \text{(model 2)} \]  
(6.40)

where \( T_s \) is the symbol period of the transmitted signal, and \( P_R \) is the average power of multipath waves and \( \sigma_T \) is the delay spread.

For model 1, each impulse \( a_{nm}^{(l)} \) is generated by the i.i.d. process while keeping \( \langle |a_{nm}^{(l)}|^2 \rangle = 1 \) for \( n = 1, 2, ..., N, m = 1, 2, ..., M \) and \( l = 0, 1, ..., L \).
6.4 Simulation Results

Although the uniform power delay profile is effective for the examination of the working mechanism of the proposed scheme, it is not realistic in indoor and outdoor propagation environments. Therefore, the exponential delay power profile will be used in order to evaluate the quantitative characteristics. The simulation conditions are summarized in Table 6.1.

Table 6.1: Simulation model for MIMO beamforming

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of array</td>
<td>linear</td>
</tr>
<tr>
<td>((M,N)) antennas</td>
<td>(4,4), (4,8)</td>
</tr>
<tr>
<td>Type of modulation</td>
<td>BPSK</td>
</tr>
<tr>
<td>Signal length</td>
<td>10(^{4}) symbols</td>
</tr>
<tr>
<td>Sampling type</td>
<td>1 sample/symbol</td>
</tr>
<tr>
<td>Input SNR</td>
<td>10dB, 20dB, 40dB</td>
</tr>
</tbody>
</table>

6.4.2 Results

First, we performed a 4 × 4 MIMO system simulation in frequency-selective fading channels environment using model 1 \((L = 7)\) and 3 delayed signals are utilized to realize the maximum output SINR at receiver based on our proposed method. The output SINRs for \(l = 0, 1, ..., 7\) obtained from the received signal \(\hat{s}(k)(t)(k = 0, 1, ..., 3)\) multiplied by the updated received vector \(\hat{w}(k)\) are shown in Figure 6.3. In order to make clear interference channels cancellation limit, we set the input SNR \(= P_S/P_N = 40dB\). Since \(A(0)\) is considered as the desired channel, the initial transmit weight vector estimated from finding the largest eigenvalue \(\lambda_{max}\) of the correlation matrix \((A(0))^HA(0)\) \((k = 0)\) is used to calculate the optimal receive and transmit weight vectors \(\hat{w}(0),_{opt}^r, \hat{w}(0),_{opt}^t\) alternately by using the iterative update weight algorithm, which has been proposed in [57]. The updated \(\hat{w}(0),_{opt}^t\) is used in the transmitter as a fixed weight vector and updated \(\hat{w}(0),_{opt}^r\) is used in the BS to detect the received signal \(\hat{s}(0)(t)\). On the other hand, based on the updated transmit weight vector \(\hat{w}(0),_{opt}^t\), the receive weight vector \(\hat{w}(k)\) corresponding to the channel \(A(k)\) is also updated to detect the delayed signal \(\hat{s}(k)(t)\). However, although the received weight vectors are estimated based on an iterative update algorithm to overcome the FSF channels, only the output SINR obtained from the received
6.4 Simulation Results

signal \( \hat{s}^{(0)}(t) \) is better than all the rest. It is clear that the degree of freedom (DOF) at both the MS \((M - 1)\) and the BS \((N - 1)\) sides are exploited to suppress \((M + N - 2 = 6)\) interferences for the received signal \( \hat{s}^{(0)}(t) \) while only the DOF of the BS side is exploited to cancel \((N - 1 = 3)\) interferences for the received signal \( \hat{s}^{(k)}(t) \).

Figure 6.4 shows the output SINRs viewing at median value of the CDF of the detected signals based on our proposed scheme for a \(4 \times 4\) MIMO system in terms of the input SNR of 40dB and 20dB. Since each the signal \( \hat{s}^{(k)}(t) \) detected is useful to improve efficiently the output SINR, the result of our proposed scheme is better than that of detection of the preceding signal only. The simulation result is shown that the improved output SINR based on our proposed method agreed very well with (6.20). However, the output SINR improvement depends on the number of BS antennas because only the DOF of the BS is used to suppress the interferences of the delayed signal \( \hat{s}^{(k)}(t)(k \neq 0) \).

In order to verify the considerably improvement of output SINR based on our proposed scheme when increasing the number of BS antennas, a \(4 \times 8\) MIMO system simulation in case of the input SNR of 40dB and 20dB are shown in Figure 6.5. It is shown that the transmit data is detected not only by using the preceding wave but also by exploiting its copies incoming the BS. On the other hand, the performance of \(4 \times 4\) and \(4 \times 8\) MIMO systems for \((L \geq 4)\) and \((L \geq 8)\), respectively, might be also improved slightly based on the proposed scheme when the input SNR is less than 40dB.

We are now going to show how our proposed schemes apply to MIMO system in FSF channels using model 2 for the delay spread of multipath waves over one symbol period. We performed a simulation for a single user case in the case of \(4 \times 4\), and \(4 \times 8\) MIMO systems with the input SNR = 10 dB and delay spread \(\sigma_T\) ranging from 0 to \(5T_s\). For comparison, \(4 \times 4\) and \(4 \times 8\) MIMO systems using the transmit and receive weight vectors obtained from finding the largest eigenvalues for the correlation matrices \((A^{(0)})^HA^{(0)}\) and \(A^{(0)}(A^{(0)})^H\), respectively, are also performed (It is called MIMO SVD). The simulation result is shown in Figure 6.6.

In Figure 6.6, both the output SINRs obtained from detection of the preceding signal and the proposed method are better than that of the MIMO SVD. Even if increasing the number of antennas at the receiver, the output SINR obtained from
6.5 Summary

$4 \times 8$ MIMO SVD is less than that of $4 \times 4$ MIMO system for detections of $\hat{s}^{(0)}(t)$ and the proposed scheme. The more delay spread $\sigma_\tau$ is larger, the more output SINR of our proposed scheme improve considerably compared to that of detection of the preceding signal $\hat{s}^{(0)}(t)$ only. Moreover, the proposed $4 \times 4$ MIMO system might be considered to be equivalent to $4 \times 8$ MIMO system for detection of the preceding signal $\hat{s}^{(0)}(t)$ only.

6.5 Summary

We have proposed spatial-temporal MIMO beamforming system for single data stream transmission. The proposed scheme is applied for a single user for frequency-selective fading channels without using the tapped delay line. The transmit and receive weight vectors of the preceding signal and the receive weight vector of each delayed signal estimated by using an iterative update algorithm are used to detect the transmitted signal incoming the base station at different time. Over which multiple detected signals are useful to maximize the output SINR compared to detection of the preceding signal only. The improvement of SINR by the proposed scheme allows that MIMO system can be suffered from interferences when the distance of transmission is made sufficiently large and the delay spread is larger than one symbol period. The providing high-bit-rate with high performance by using our proposed scheme without using the tapped delay lines is possible in MIMO frequency-selective fading channels.
6.5 Summary

(a)

(b)
Figure 6.3: Distribution function of SINR for a $4 \times 4$ MIMO system: (a) CDF for $\hat{s}^{(0)}(t)$; (b) CDF for $\hat{s}^{(1)}(t)$; (c) CDF for $\hat{s}^{(2)}(t)$; (d) CDF for $\hat{s}^{(3)}(t)$
Figure 6.4: Median value of SINR vs. the number of interferences for a $4 \times 4$ MIMO system: (a) $\text{SNR}_{\text{in}} = 40\text{dB}$; (b) $\text{SNR}_{\text{in}} = 20\text{dB}$
6.5 Summary

Figure 6.5: Median value of SINR vs. the number of interferences for a 4×8 MIMO system: (a) $\text{SNR}_{\text{in}} = 40\text{dB}$; (b) $\text{SNR}_{\text{in}} = 20\text{dB}$
Figure 6.6: Average SINR of MIMO system as a function of the delay spread $\sigma$. 

4x4 and 4x8 MIMO Beamforming ($\text{SNR}_{in} = 10\text{dB}$) 

- 4x8 Proposed scheme 
- 4x8 MIMO $p^{(0)}$ 
- 4x4 Proposed scheme 
- 4x4 MIMO $p^{(0)}$ 
- 4x8 MIMO SVD 
- 4x4 MIMO SVD
Chapter 7

Conclusion and Future Work

This chapter summarizes the results of this work and presents open topics for the future research. Finally, conclusion of is included.

7.1 Conclusions

In recent years, MIMO antennas systems are used in fading environments to exploit an enormous capacity advantage. It is regarded as one of the most promising technologies to improve performance and coverage of system. Several techniques have been studied successfully for flat fading environment but frequency-selective fading channels. In order to solve the MIMO frequency-selective fading channels, spatial multiplexing OFDM has been proposed to transmit multiple independent streams simultaneously. However, the performance of system is degraded by increasing the distance of transmission. And besides, the decision feedback equalization technique, where MIMO antennas systems equipped with TDL structure, will be one of the topics research for mitigating the frequency-selective fading channels and maximize the performance of system. However, they so far still suffer from computational complexity, compact and low-cost hardware. Meanwhile, the determination of optimal transmit and receive weight vectors for MIMO beamforming system using a single data stream transmission is one of the simple and sufficient solutions for frequency-selective fading channels and maximize the output SINR at the BS. According to the numerical and simulation results shown in this work, the following concluding remarks are obtained.

- The optimal transmit and receive weights determination scheme for MIMO
beamforming for single data stream transmission in frequency-selective fading channels has been analyzed and developed. Through studying the iterative weight update algorithm for the transmitter and receiver in the MIMO beamforming, we have successfully proposed the cancellation of \((M+N-2)\) delayed channels for MIMO system with \(M\) and \(N\) antenna elements at mobile and base stations, respectively. Using this method, MIMO beamforming system can effectively mitigate multipath fading in frequency-selective fading channels with low computational complexity compared to MIMO system using TDL structure.

- Deriving from the weights determination scheme, MIMO beamforming using single data stream transmission for multiuser system has been proposed. Thereon, a maximum number of interference channels, which could be eliminated in multiuser system, is analyzed in two cases of the receive weight vectors optimization only and iterative update of both transmitter and receiver weight vectors optimization. For the first case, which is characterized by simple scheme, shown good performance for propagation channel with the preceding channel of the desired user has few its own interference channels, while the second case shown better performance but more sophisticated scheme.

- Since the preceding signal is considered as the desired signal and the subsequent signals are referred to as interferences reflected at increasing the distance of transmission, which described in Chapter 6. However, the more delayed signals increase the more performance of system is degraded. In order to improve the output SINR at the BS, the spatial-temporal MIMO beamforming for single data stream transmission has been analyzed and developed. Using this method, not only the preceding signal but also several delayed versions of the preceding signal are detected at the receiver. Over which, multiple detected signals are useful to maximize the performance of system compared to that of detection of the preceding only. This improvement of performance is useful for MIMO systems to complement the high-quality in broadband wireless communication when the distance of transmission is made sufficiently large.
7.2 Future Work

Referring to the above conclusions, there are several areas of this work which could be extended for future research.

- MIMO beamforming for multiple data streams transmission in frequency-selective fading channels is a challenging topic that increases the capacity compared to the MIMO beamforming for single data stream transmission while maximizing the SINR. It is expected to realize extremely high capacity communication system.

- MIMO beamforming for multi-carrier transmission is expected to improve both capacity and performance of system compared with MIMO beamforming for single carrier transmission in frequency-selective fading environment.

- MIMO propagation channel measurement and characterization at 5GHz with the expected results based on the proposed researches will be of great benefits for implementing wideband MIMO frequency-selective fading channels.

- Novel MIMO data transmission scheme in fast fading environment for vehicular communications is the next research that contributes to the new-generation wireless access system such as MIMO cellular system, intelligent transportation system (ITS), and ad hoc networks.
Appendix A

Determination of Transmit Weight Vector $w_{t,i}^{(opt)}$ for the $i$th User

In the appendix, we will explain the transmit weight vector determination.

Starting from Eq.(5.11), we rewrite the output SINR for the $i$th user expression as

$$\Gamma(w_{t,i}; w_{r,j} (j = 1, ..., Q, j \neq i)) = \frac{w_{t,i}^H A_i^{(o)} w_{r,i} w_{r,i}^H (A_i^{(o)})^H w_{t,i}}{\tilde{R}_{\text{inf}} + \gamma_i w_{r,i}^H w_{r,i}} \quad (A.1)$$

where

$$\tilde{R}_{\text{inf}} = \sum_{l=1}^{L_i-1} w_{t,i}^H A_{i}^{(l)} w_{r,i} w_{r,i}^H (A_{i}^{(l)})^H w_{t,i}$$

$$+ \sum_{j=1, j \neq i}^{Q} \sum_{l=0}^{L_j-1} w_{t,j}^H A_{j}^{(l)} w_{r,j} w_{r,j}^H (A_{j}^{(l)})^H w_{t,i} \quad (A.2)$$

We use the Lagrange multiplier method under condition of $w_{t,i}^H w_{t,i} = 1$. Specifically

$$\phi_i = \Gamma(w_{t,i}; w_{r,j} , (j = 1, ..., Q, j \neq i)) + \lambda_i (1 - w_{t,i}^H w_{t,i}) \quad (A.3)$$

where $\lambda_i$ represents the Lagrange multiplier.

Taking the derivative of (A.3) with respect to $w_{t,i}$ and setting it to zero.

$$\frac{\partial \phi_i}{\partial w_{t,i}} = \frac{\partial \Gamma(w_{t,i}; w_{r,j} , (j = 1, ..., Q, j \neq i))}{\partial w_{t,i}} + \frac{\lambda_i (1 - w_{t,i}^H w_{t,i})}{\partial w_{t,i}} = 0 \quad (A.4)$$
And as a consequence, we have

\[
\frac{[A^{(0)}_i]^H w_{r,i} H w_{r,i} A^{(0)}_i w_{t,i}]}{(R_{inf} + \gamma_i w_{r,i} w_{r,i})^2} \left[ \sum_{l=1}^{L_i} (A^{(l)}_i)^H w_{r,i} H w_{r,i} A^{(l)}_i w_{t,i} \right] (w_{t,i} (A^{(0)}_i)^H w_{r,i} w_{r,i} A^{(0)}_i w_{t,i}) - \lambda_i w_{t,i} = 0 \quad (A.5)
\]

In order to calculate the value of \( \lambda_i \), (A.5) is multiplied by weight vector \( w_{t,i}^H \) with the condition \( \|w_{t,i}\| = 1 \), then we have

\[
\lambda_i = \frac{w_{t,i}^H (A^{(0)}_i)^H H w_{r,i} H w_{r,i} A^{(0)}_i w_{t,i} \sum_{l=1}^{L_i} \sum_{j=1,j \neq i}^{L_j} w_{t,j} (A^{(l)}_j)^H H w_{r,i} H w_{r,i} A^{(l)}_j w_{t,j}}{(R_{inf} + \gamma_i w_{r,i} w_{r,i})^2} + \gamma_i w_{r,i} w_{r,i} \quad (A.6)
\]

Substituting (A.6) into (A.5), and since \( w_{r,i}^H A^{(0)}_i w_{t,i} \) is a scalar, we have

\[
w_{t,i} = \frac{R_{inf} + \gamma_i w_{r,i} w_{r,i} R_{nt,i}^{-1} (A^{(0)}_i)^H H w_{r,i}}{w_{t,i}^H (A^{(0)}_i)^H H w_{r,i}} (A.7)
\]

where \( R_{nt,i} \) is defined in the Chapter 5, and is given by

\[
R_{nt,i} \equiv \sum_{l=1}^{L_i-1} (A^{(l)}_i)^H w_{r,i} H w_{r,i} A^{(l)}_i + \sum_{j=1,j \neq i}^{L_j-1} \sum_{l=0}^{L_j} w_{t,j} (A^{(l)}_j)^H H w_{r,i} H w_{r,i} A^{(l)}_j w_{t,j} + \gamma_i I \quad (A.8)
\]

Let us define

\[
\xi = \frac{R_{inf} + \gamma_i w_{r,i} w_{r,i}}{w_{t,i}^H (A^{(0)}_i)^H H w_{r,i}} \quad (A.9)
\]

Equation (A.7) can be written as

\[
w_{t,i} = \xi R_{nt,i}^{-1} (A^{(0)}_i)^H H w_{r,i} \quad (A.10)
\]

Without detailed calculation of \( \xi \), the optimal transmit weight vector for the \( i \)th user can be normalized and given by

\[
w_{t,i}^{(opt)} = \frac{R_{nt,i}^{-1} (A^{(0)}_i)^H H w_{r,i}}{\| R_{nt,i}^{-1} (A^{(0)}_i)^H H w_{r,i} \|} \quad (A.11)
\]
REFERENCE


List of Original Publication Related to the Dissertation

Journal Paper


3. Huy Hoang Pham, Tetsuki Taniguchi, and Yoshio Karasawa, “Spatial-Temporal Adaptive MIMO Beamforming for Frequency-Selective Fading Channels,” IEICE Transactions on Communications. (Under 1st Review) (Related to the content of Chapter 6)

International Conference and Presentations

1. Huy Hoang Pham, Tetsuki Taniguchi, and Yoshio Karasawa, “MIMO Beamforming for High-Bit-Rate Transmission over Frequency-Selective Fading Channels,” The IEEE International Symposium on Spread Spectrum Techniques and Applications (ISSSTA ’04), Sydney, Australia, September 2004. (Related to the content of Chapter 4)


5. Huy Hoang Pham, Tetsuki Taniguchi, and Yoshio Karasawa, “Spatial-Temporal Adaptive MIMO Beamforming for High-Quality Transmission over Frequency-Selective Fading Channels,” The IEEE Advanced International Conference on Telecommunications (AICT’06), Guadeloupe, French Caribbean 2006. (Submitted) (Related to the content of Chapter 6)

Other Conference Papers and Presentations


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Huy Hoang Pham was born in Thai Binh, Vietnam on 16th February 1976. He received B.E degree from Ho Chi Minh University of Technology, Vietnam in 1999 and M.S degree from the University of Electro-Communications, Tokyo, Japan in 2003. He is currently a Ph.D. student in Department of Electronic Engineering, The University of Electro-Communications, Tokyo, Japan. His research interests are in the areas of adaptive array antenna, space-time adaptive beamforming, DS-CDMA for FDD system, flat and frequency selective fading channels, multiple-input multiple-output and multiuser systems.