SUMMARY In this paper, we propose a multiuser detection (MUD) scheme for space-time block coded orthogonal frequency division multiplexing (STBC-OFDM) systems. We derive the optimum weight matrix used to decode simultaneously signals from active multiple access users using the minimum mean square error (MMSE) multiuser detection method. The proposed scheme provides good performance over different models of the frequency selective fading channel. It is also to show that if the length of the cyclic prefix is larger than that of the channel, the performance of the detector depends on only the total energy extracted from multipath components but not the employed channel model, the number of multipath components or the delay of each multipath component.

key words: space-time code, transmit diversity, OFDM, multipath fading, multiuser detection

1. Introduction

Next generation of wireless mobile communications requires the reliable transmission of the high-rate data under various types of channels. It is well known that multipath fading is an inherent obstacle which affects the quality of data transmission over the wireless channels. Diversity techniques, particularly the time and space diversity, have been realized as an effective method for mitigating multipath fading [1]. Recently, these two diversity techniques were combined together to generate the so-called space-time codes (STCs) [2]–[4]. It was shown that space-time codes can provide high-order diversity gain, which helps to increase the transmission data rate without requiring the bandwidth expansion [2]–[4], over the flat fading channels. However, when the transmission is done over the broadband channels, the frequency selective fading channel, further processing techniques or efficient modulation schemes should be used in conjunction with space-time codes to combat the inter-symbol interference (ISI) due to the channel delay spread. To this direction, multi-carrier modulation, namely, the orthogonal frequency division multiplexing (OFDM), is a promising technique as it can convert the frequency selective channel into a number of flat fading channels which can be easily handled using conventional equalization techniques. The ISI then can be completely removed if the cyclic prefix (CP) or guard interval (GI) length is selected such that its length is larger than that of the channel [5]. Due to these attractive merits, OFDM has been accepted as standards for the digital audio broadcasting (DAB), the digital video broadcasting (DVB), high performance local area networks (HIPERLAN), the IEEE 802.11.a wireless local area network (WLAN) [5] and the IEEE 802.16.a wireless metropolitan area network (WMAN) [6]. It is also being considered a candidate for the wireless mobile broadband access within the IEEE 802.20 standard [7].

The combination of space-time codes and OFDM thus promises large advantages and has been studied widely by many researchers [8]–[14]. In [8]–[11], the authors showed that OFDM could be combined well with space-time codes using maximum likelihood (ML) based detector to improve the system performance over the frequency selective channel. The applications of other detection techniques such as spatial prewhitening, successive interference cancelation and decision directed for detecting the STC-OFDM signals were also considered in [12] and [13]. However, all these works [8]–[13] were done only for single user channel. Recently, a method of detecting STC-OFDM signals for the channels with co-channel interferences (CCIs) based on the beamforming approach, i.e., nulling out other co-channel user signals, has been proposed in [14]. The practical wireless mobile communication systems, however, require support of multiuser communications meaning that instead of eliminating the signals from co-channel multiple access users, it is necessary that the base station be able to detect simultaneously the signals from all active users. Multiuser detection (MUD) techniques, therefore, should be used at the base station to fulfill this task.

We investigate in this paper the problem of multiuser detection for the space-time block coded [2], [4] OFDM (STBC-OFDM) systems. In particular but without loss of generality, we shall consider the case of the simple Alamouti’s space-time block code as it is the only scheme which can provide full rate and full diversity for any signal constellations. Among available multiuser detection techniques [5], [15] the most popular minimum mean square error (MMSE) method is chosen since it can provide good performance and can be readily implemented using standard adaptive algorithms. This MMSE multiuser detector is based on our previously proposed beamforming approach for STBC systems in [16]. The main contributions of the paper include a linear signal processing scheme which converts the problem of the STBC-OFDM systems into that of conventional systems and the derivation of the detector weight ma-
tix based on the MMSE method. We first show that by using our processing scheme a $Q$-user STBC-OFDM system using Alamouti’s encoding scheme can be expressed in a similar model of a $2Q$-user single input multiple output (SIMO)-OFDM systems. It is then to demonstrate that the MMSE method can be well applied to the scheme to decouple simultaneously signals from active multiple access users. Performance results using computer simulation show that the proposed detector can provide good performance under different models of frequency selective fading channels.

The remainder of this paper is organized as follows. Section 2 describes the configuration and the signal model of the proposed detector for STBC-OFDM. Performance analysis is presented in Sect. 3 and simulation results are shown in Sect. 4. Finally, Sect. 5 concludes the paper.

2. MMSE Multiuser Detector for STBC-OFDM

2.1 System Model

In this section we introduce an MMSE-based multiuser detector for STBC-OFDM. Consider an up-link multiuser STBC-OFDM communication system as illustrated in Fig. 1, in which $Q$ active transmitters (Tx) using STBC-OFDM to transmit simultaneously their signals to the receiver (Rx) over the frequency selective fading channels. Using the Alamouti STBC [2] each block of $K$ data symbols of user $q \in \{1, 2, \ldots, Q\}$ is space-time encoded (STE) using the following encoding matrix

$$X[q][k] = \begin{bmatrix} S[q][k][1] & S[q][k][2] \\ S[q][1,2] & S[q][2,2] \end{bmatrix} = \begin{bmatrix} X[q][k] & X[q][k] \\ X[q][k] & X[q][k] \end{bmatrix},$$

where $k \in \{0, 1, \ldots, K-1\}$; $S[q][k] \in C$ are the transmit symbols from antenna $n \in \{1, 2\}$ of user $q$ at time slot $t \in \{1, 2\}$; $\prescript{*}{}$ denotes the complex conjugation operation; $C$ is the signal constellation. For notational convenience the block index has been omitted. The coding rule in (1) is explained for user $q$ as follows. At time slot $t = 1$, the first antenna transmits $X[q][1][k]$ while the second antenna transmits $X[q][2][k]$. At the next time slot $t = 2$, the first antenna transmits $-X[q][1][k]$ while the second antenna transmits $X[q][2][k]$. Next, the inverse fast Fourier transform (IFFT) is applied to convert each $K$-length block of symbols into the time domain and the last $C$ symbols are copied and appended into the front of each block as the cyclic prefix (CP). The length of the OFDM symbol is thus $(K + C)T_s$, where $K$ is the IFFT size and $T_s$ is sampling interval which also means the symbol duration for the symbol-spaced signals considered in this paper. The transmitted signals from the $n$th antenna of user $q$ are given by

$$x[n][k] = \sum_{k=0}^{K-1} S[n,q][k]e^{j2\pi n (k-\alpha)},$$

where $\alpha$ denotes the delay in symbols at each block.

The receiver is assumed to have $M$ antennas, which forms a $2 \times M$ multiple input multiple output (MIMO) channel between each transmitter and the receiver. It is also assumed that antenna elements are spaced large enough such that fading is spatially uncorrelated at the base station antenna array. The multipath channels $h[n,m][k]$ between the receiver antenna $n$ of user $q$ and receive antenna $m$ can be described by $P$ resolveable multipath components $p = 0, 1, \ldots, P - 1$, which can be expressed for the case of symbol-spaced signals as

$$h[n,m][k] = \sum_{p=0}^{P-1} a[n,m,p] \delta[l - p].$$

Under quasi-static condition, i.e., the channels are constant over some block of arbitrary length: $a[n,m,1][p] = a[n,m,2][p] \equiv a[n,m,p]$ and $h[n,m,1][k] = h[n,m,2][k] \equiv h[n,m,k]$. Then the received signals at receive antenna $m$ and time $t$ can be expressed as

$$y[n,m][k] = \sum_{q=1}^{Q} \sum_{l=0}^{P-1} a[n,m,p] x[n,q][l-p] + z[n,m][k],$$

where the path index $p$ can also be interpreted as the channel delay in units of symbol intervals; $z[n,m][k]$ are the local complex Gaussian noise samples at antenna $m$ and time $t$, which are assumed to be independent and identically distributed (i.i.d.) with mean 0 and variance $\sigma_z^2$. Note that we have assumed perfect carrier frequency synchronization in (4) to simplify our further analysis. As OFDM is very sensitive to the carrier frequency offset, the practical systems, however, should employ effective synchronization methods such as those described in [17] and [18] to mitigate the intercarrier interference (ICI). The signals $y[n,m][k]$ are then OFDM demodulated and put through a multiuser detector (MUD), which is abbreviated as OFDM-MUD, to detect the transmit signal from each transmitter.

If the CP length $C$ is chosen such that $C \geq P - 1$, then after discarding the CP and performing FFT the demodulated signals in the frequency domain are given by

$$Y[n,m][k] = \sum_{q=1}^{Q} \sum_{n=1}^{2} \sum_{p=0}^{P-1} a[n,m,p] x[n,q][l-p] + Z[n,m][k].$$

Fig. 1 An illustration of multiuser STBC-OFDM systems.
where $H_{mn}^{(q)}[k]$ are the channel frequency response from the $n$th transmit antenna of user $q$ to the $m$th receive antenna during time slot $t \in \{1, 2\}$. Denote

$$H_{mn}^{(q)[p]} = \left[ a_{mn}^{(q)[0]} \ a_{mn}^{(q)[1]} \ldots a_{mn}^{(q)[P-1]} \right]^H$$

$$\epsilon_{p}^{k} = \left[ 1 \ e^{-j2\pi \alpha_{p}} \ldots e^{-j2\pi (P-1) \alpha_{p}} \right]^T$$

where $(.)^T$ and $(.)^H$ represents the vector/matrix transpose and the Hermitian operation, respectively. For the tolerable leakage the channel frequency response $H_{mn}^{(q)}[k]$ can be given by [12]

$$H_{mn}^{(q)}[k] = \sum_{p=0}^{P-1} a_{mn}^{(q)[p]} e^{-j2\pi \alpha_{p}} = h_{mn}^{(q)}H \epsilon_{k}^{p}.$$  

The received signals $Y_{m,t}[k]$ are then parallel-to-serial (P/S) converted into 2 streams $Y_{m,1}[k]$ and $Y_{m,2}[k]$, and then the complex conjugation is applied to $Y_{m,2}[k]$ as shown in Fig. 2.

2.2 Proposed Configuration

We now present the theoretical model of the proposed configuration of the MMSE multiuser detector for STBC-OFDM. Configuration of the detector is shown in Fig. 2. We begin describing the configuration by attempting to rewrite (4) into the vector form. Let us define two channel vectors

$$h_1^{(q)}[k] = \left[ H_{11}^{(q)}[k] \ H_{21}^{(q)}[k] \ldots H_{M1}^{(q)}[k] \right]^T$$

$$h_2^{(q)}[k] = \left[ H_{12}^{(q)}[k] \ H_{22}^{(q)}[k] \ldots H_{M2}^{(q)}[k] \right]^T$$

and similarly, the signal and noise vectors as

$$x^{(q)}[k] = \left[ X_1^{(q)}[k] \ X_2^{(q)}[k] \right]^T$$

$$y_1[k] = \left[ Y_{11}[k] \ Y_{21}[k] \ldots Y_{M1}[k] \right]^T$$

$$y_2[k] = \left[ Y_{12}[k] \ Y_{22}[k] \ldots Y_{M2}[k] \right]^T.$$ 

We then stack $y_1[k], y_2[k]$ and $h_i^{(q)}[k], i \in \{1, 2\}$ as

$$y[k] = \left[ y_1[k] \ y_2^H[k] \right]^T$$

$$z[k] = \left[ z_1[k] \ z_2^H[k] \right]^T.$$  

where

$$H^{(q)}[k] = \left[ h_1^{(q)[k]} h_2^{(q)[k]} \right]$$

and

$$x[k] = \left[ x^{(1)}[k] \ x^{(2)}[k] \ldots x^{(Q)}[k] \right]^T$$

$$H[k] = \left[ H^{(1)}[k] \ H^{(2)}[k] \ldots H^{(Q)}[k] \right].$$

Now the system equation (4) can be conveniently expressed in the vector form using (14), (15), (17) and (18) as

$$y[k] = H[k]x[k] + z[k].$$

Since the carrier index $k$ is common, for the notational convenience, we shall henceforth ignore it in our expressions, i.e.

$$y = Hx + z.$$  

Note that the system equation (20) is written in a similar form with that for the multiuser OFDM system with receive diversity or SIMO-OFDM in [5]. The difference is that in our case of STBC-OFDM each user is equipped with 2 antennas, thus each element of $x$ can be considered the transmitted signal from one of $2Q$ antennas, and similarly each column of $H$ can also be seen as the channel frequency responses from one transmit antenna to the receiver antennas. This property allows us to use conventional multiuser detection techniques for SIMO systems to decouple each STBC-OFDM user signal. In this paper, we use the most popular linear multiuser method, namely, MMSE, for detecting transmitted symbols from the $Q$ users. Application of other multiuser detection methods [5], [15] to (20) can be done in a similar way.

Given (20) our objective is to derive a detector with the weight matrix $W \in C^{2M \times 2Q}$

$$W = \left[ w_1^{(1)} \ w_2^{(1)} \ldots w_1^{(Q)} \ w_2^{(Q)} \right]$$

used to decouple each user transmitted signals $X_1^{(q)}$ and $X_2^{(q)}$ via the linear combining

$$\hat{x} = W^Hy,$$  

where

$$\hat{x} = \left[ \hat{X}_1^{(1)} \ \hat{X}_2^{(1)} \ldots \hat{X}_1^{(Q)} \ \hat{X}_2^{(Q)} \right]^T.$$  

For the binary phase shift keying (BPSK) signals, the estimated symbols are decided using the signum function as

$$\hat{x} = \text{sgn}([\hat{x}] = \text{sgn}[WHy].$$  

Without loss of generality let us consider the case of user $v$. Our task now is to derive two vectors $w_1^{(v)}$ and $w_2^{(v)}$ used to decouple the desirably transmitted symbols $X_1^{(v)}$ and
\[ X_2^{(v)} \] \. The MMSE solutions for the detector are defined as
\[
\begin{align*}
\hat{w}_1^{(v)} &= \arg \min_{w_1^{(v)}} E \left\{ |X_1^{(v)} - w_1^{(v)} y| \right\} \\
\hat{w}_2^{(v)} &= \arg \min_{w_2^{(v)}} E \left\{ |X_2^{(v)} - w_2^{(v)} y| \right\}
\end{align*}
\] (25)
(26)
where \( w_1^{(v)} \) and \( w_2^{(v)} \) are two vectors corresponding to the \((2v-1)\)-th and \(2v\)-th column of the weight matrix \( W \), respectively; similarly \( X_1^{(v)} \) and \( X_2^{(v)} \) are, respectively, the \((2v-1)\)-th and \(2v\)-th elements of the input vector \( x \); and \( E[\cdot] \) represents the expected value. Taking derivative of the arguments of (25) and (26) with respect to \( w_1^{(v)} \) and \( w_2^{(v)} \), respectively, then equating them to zero yields (see Appendix for detailed derivation)
\[
\begin{align*}
\hat{w}_1^{(v)} &= (H \Lambda H^H + \sigma_z^2 I_{2M})^{-1} \hat{h}_1^{(v)} \\
\hat{w}_2^{(v)} &= (H \Lambda H^H + \sigma_z^2 I_{2M})^{-1} \hat{h}_2^{(v)}
\end{align*}
\] (27)
(28)
where \( I_{2M} \in \mathbb{C}^{2M \times 2M} \) is an identity matrix, i.e., with all zeros except ones in the diagonal,
\[
\begin{align*}
\hat{h}_1^{(v)} &= \hat{z}_v^2 \hat{h}_1^{(v)T} \\
\hat{h}_2^{(v)} &= \hat{z}_v^2 \hat{h}_2^{(v)T} - \hat{h}_1^{(v)H} \\
\hat{z}_v^2 &= E[|X_v^{(v)}|^2], \, t \in \{1, 2\}, \text{is the average power of the transmitted signals, and}
\end{align*}
\] (29)
(30)
\[ \Lambda \equiv \begin{bmatrix} \Lambda_1 & 0 & \cdots & 0 \\
0 & \Lambda_2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \Lambda_Q \end{bmatrix} \] (31)
is the diagonal power matrix with \( \Lambda_q = \hat{z}_v^2 I_2 \). Note that the term
\[
R_{yy}^{-1} \equiv (H \Lambda H^H + \sigma_z^2 I_{2M})^{-1}
\] (32)
is in fact the inverse of the covariance matrix of the received signal \( y \), i.e., \( R_{yy} = E(yy^H) \). Therefore, the weight vectors \( w_1^{(v)} \) and \( w_2^{(v)} \) can be expressed in the well-known form of the Wiener-Hopf solution [19] as
\[
\begin{align*}
\hat{w}_1^{(v)} &= R_{yy}^{-1} \hat{h}_1^{(v)} \\
\hat{w}_2^{(v)} &= R_{yy}^{-1} \hat{h}_2^{(v)}
\end{align*}
\] (33)
(34)
Replacing \( \hat{w}_t^{(v)}, \, t \in \{1, 2\}, \) in (27) and (28) into (21), then using (33) and (34), we obtain the weight matrix
\[
W = (H \Lambda H^H + \sigma_z^2 I_{2M})^{-1} H \Lambda
\] (35)
which is expressed in the same form with that of the SIMO-OFDM system in [5].

The estimated symbols are then given using (24). Specifically, for the user of interest \( v \) the estimates \( \hat{X}_1^{(v)} \) and \( \hat{X}_2^{(v)} \) are decided as
\[
\begin{align*}
\hat{X}_1^{(v)} &= \text{sgn} \left\{ \hat{w}_1^{(v)H} y \right\} \\
\hat{X}_2^{(v)} &= \text{sgn} \left\{ \hat{w}_2^{(v)H} y \right\}
\end{align*}
\] (36)
(37)

3. Performance Analysis

3.1 MSE Performance

Define the estimation error vector \( e \) as
\[
\epsilon = x - \hat{x} = x - W^H y.
\] (38)

Then the estimation error covariance matrix is given by
\[
R_\epsilon = E[\epsilon \epsilon^H].
\] (39)

Replacing (38) into (39) while noting that \( E[xx^H] = \Lambda, E[nn^H] = \sigma_z^2 I_{2M} \), and \( E[xn^H] = 0 \), we have
\[
R_\epsilon = \Lambda - R_{yy}^H W - W^H R_{yy} + W^H R_{yy} W
\] (40)
where
\[
R_{yy} = E[yy^H] = H \Lambda.
\] (41)

Using (35) the estimation error covariance matrix \( R_\epsilon \) becomes
\[
R_\epsilon = \Lambda - \Lambda^H H^H W = \Lambda - R_{yy}^H W
\] (42)
The total average MSE of all \( Q \) users is then given by [5]
\[
\text{MSE} = \frac{1}{2Q} \text{trace}(R_\epsilon).
\] (43)

For the user of interest \( v \), the MSE\(^{(v)}\) is the average of MSEs for detecting \( \hat{X}_1 \) and \( \hat{X}_2 \), i.e., the average of the \((2v-1)\)-th and \(2v\)-th diagonal elements of \( R_\epsilon \). However, due to the symmetrical consideration the MSEs are expected to be the same and thus we shall need to calculate only one of the MSEs, for example MSE\(^{(v)}_1 \) [5]
\[
\text{MSE}^{(v)} = \text{MSE}^{(v)}_1 = \hat{z}_v^2 \left\{ 1 - H_1^{(v)H} \hat{w}_1^{(v)} \right\}
\] (44)
where \( H_1^{(v)} \) denotes the first column of \( H^{(v)} \), i.e., the \((2v-1)\)-th column of \( H \). Note that the same expression for MSE in (44) can be obtained by replacing (27) into (A-10).

3.2 SINR Performance

The signal-to-interference-plus-noise ratio (SINR) of the detector can be calculated using the same method for adaptive antennas. From (23) define the signal vector at the detector’s output for user \( q \) as
\[ \hat{x}^{(q)} = \left[ \hat{x}_1^{(q)}, \hat{x}_2^{(q)} \right]^T. \]  
(45)

The cross-correlation between the transmit signal \( x^{(q)} \) and the output signal \( \hat{x}^{(q)} \) is given by [16]

\[ \rho^{(q)} = \frac{E\{x^{(q)}\hat{x}^{(q)*}\}}{\sqrt{E\{|x^{(q)}|^2\}E\{|\hat{x}^{(q)}|^2\}}}. \]  
(46)

The output SINR for user \( q \) is then given via the cross-correlation \( \rho \) as [16]

\[ \text{SINR}^{(q)} = \frac{|\rho^{(q)}|^2}{1 - |\rho^{(q)}|^2}. \]  
(47)

4. Simulation Results

4.1 System Parameters

We explore the performance of the proposed MMSE multiuser detector for BPSK signalling using computer simulation. In our simulation, the total available bandwidth \( B \) is assumed to be 1 MHz, which is divided into \( K = 256 \) sub-channels using IFFT. This results in 3.9 kHz sub-channels and the effective OFDM blocks of 256 µs. Each OFDM block is appended with a cyclic prefix of length \( C = 40 \) µs which is assumed to be larger than the channel length \( (P - 1) \) for perfect ISI mitigation. The transmitters are each equipped with \( N = 2 \) transmit antennas, while the base station has \( M = \{2, 4\} \) antennas, which respectively results in \( 2 \times 2 \) and \( 2 \times 4 \) MIMO channels. The channels \( h_{mn}^{(q)}[l] \) between each base station antenna and transmit antenna of user \( q \) are assumed broadband Rayleigh fading and are modeled using 2-path, and the exponential channel models illustrated in Fig. 3. We also assume that all the channels are quasi-static and are \( a \) priori known or perfectly estimated. To simulate the fading we use the Jakes model [1] with the Doppler frequency of 25 Hz which corresponds to the transmitter using 900 MHz carrier frequency and moving at speeds of 30 km/h. The transmit power from each antenna of all users is normalized to 1/2 of the transmit power. Throughout the simulations, the MSE and SINR performance are computed using (44) and (47), while BER performance is obtained via Monte-Carlo simulation. In order to validate simulation results, we use the property that the performance of a \( 2 \times M \) MIMO-STBC system is 3 dB worse than that of a \( 1 \times 2M \) SIMO maximum ratio combining (MRC) system due to power normalization [2]. The validation is done for the case of one user in Sect. 4.2.2 below.

4.2 Results

4.2.1 BER Performance under Different SIR Conditions

In the first simulation, we explore the performance of the MMSE detector for different values of the signal-to-interference-ratio (SIR), or leakage coefficients [15]. The channel taken for simulation is a \( 2 \times 2 \) MIMO with \( Q = 2 \) users. Both the users are assumed to have the same channel delay profile which is modeled using the 2-path equal power model shown in Fig. 3(a). Three values of SIR, namely, \( \text{SIR} = \{-10, 0, 10\} \) dB, were taken for consideration. BER performance of the detector for the three values of SIRs is shown in Fig. 4. It is clearly seen from the figure that the detector provides good BER performance, particularly, at high values of SIR and \( E_b/N_0 \).

4.2.2 SINR, MSE and BER Performance for Different Number of Users

Figures 5(a), 5(b), and 5(c) illustrate, respectively, the SINR, MSE and BER performance of the detector with different number of active users in a \( 2 \times 4 \) MIMO channel. The channel delay profile for all users is the 2-path channel model used in the previous simulation. All users are assumed to have equal power, i.e., SIR = 0 dB. Users are numerically added one by one to the system while user 1 is
result, the performance of one user case is expected to be the same with that of a single user $1 \times 2M$ SIMO MRC system under flat fading channel. It is known that under flat fading channel without CCI, an $1 \times 2M$ SIMO MRC can ideally produce the output SINR: $\text{SINR}[\text{dB}] = G_{\text{array}}[\text{dB}] + \text{SNR}[\text{dB}]$, where $G_{\text{array}}$ and SNR represent the array gain and the input signal to noise ratio, respectively [20]. In the current case for $1 \times 8$ SIMO MRC system, at SNR=0 dB, for example, the maximum achievable SINR for is $\text{SINR}[\text{dB}] = 10 \log_{10}(8) + 0 \approx 9$ dB. This value matches well with that in Fig. 5(a). The corresponding MSE performance for this case is [5],[19]; $\text{MSE}^{(v)} = \frac{\xi^2_v}{1 + \text{SINR}^{(v)}}$. Similarly, the BER result for one user in Fig. 5(c) can be confirmed to be the same with that using the theoretical value of a $1 \times 8$ SIMO MRC system given in section 14.4 of [17].

From the figures it is immediately realized that the performance of the detector gradually decreases as the number of active users increases. The reason for the performance degradation is due to the reduced number of degree of freedom (DOF). An $M$-antenna array has a maximum number of $(M - 1)$ DOF. For a system with a small number of users and thus having a large available number of DOF, the detector can use the available DOF to adjust its weight matrix $W$ to simultaneously exploit the channel diversity, minimize local noise and mitigate other CCIs. Principally, it requires one DOF to suppress one CCI (see [16] and the references therein for detailed discussion for the case of STBC systems, and thus when the number of users increases the available DOF decreases, sparing the detector with less DOF. Obviously, with the reduced DOF the detector cannot adjust its weights to fulfill the three simultaneous tasks as good as in the case with more DOF. The performance is hence expected to degrade with more users. It is also well known that the MMSE detector is a suboptimal detector with respect to the capability of minimizing probability of error [5],[15]. In fact the MMSE detector minimizes the MSE rather than the BER. Under the BER minimization criterion and with a large number of users, the ML detector has been known to provide better performance than an MMSE [21]. However, the ML detector requires excessive complexity, hence is not as attractive as the MMSE [5].

Comparing two lines with down triangle marks (▽) in Figs. 4 and 5(c) we can see that keeping the same number of users ($Q = 2$) in the system while increasing the number of receive antennas (from $M = 2$ in Fig. 4 to $M = 4$ in Fig. 5(c)) improves the performance significantly. This is apparent since increasing the number of receive antennas also means increasing the system diversity order, and thus the detector benefits additional diversity gain to enhance performance.

4.2.3 BER Performance under Different Fading Channel Models

The BER performance for a 2-user $2 \times 2$ MIMO and $2 \times 4$ MIMO STBC-OFDM evaluated in the 2-path and exponential model channels is shown in Fig. 6. User 1 is assigned with the 2-path channel (Fig. 3(a)) while user 2 with the 6-
path exponential channel (Fig. 3(b)) model, arbitrarily. The channel delay profile of user 2 is normalized such that the total power from all its paths equal that of the user 1. It can be seen that both the users have the same BER performance. As the channels were arbitrarily assigned to the 2 users, simulation results for the case in which user 1 is with the 6-path exponential channel while user 2 with the 2-path channel are expected to be the same with those in the previous case. Since we assume in this paper that the CP is longer than the channel length, it is possible to conclude that as long as the CP is selected to be longer than the channel length, the performance of the MMSE detector depends only on the total energy extracted from multipath components rather than the used channel model, the number of multipath components or the delay of each multipath component. For the case in which the CP length is smaller than that of channel, this conclusion may not be true as the performance of the detector depends significantly on excessive degree of multipath delay over the CP.

5. Conclusion

In this paper we have proposed an MMSE multiuser detector for STBC-OFDM. It was shown that use of large number of receive antennas can help to improve the performance of the detector thanks to the benefit from the increased order diversity. We have also shown that the performance of the detector is degraded with increased number of users due to its sub-optimality and reduced DOF. Finally, the performance of the detector was shown to depend on only the total energy extracted from multipath components but not on the used channel model, the number of multipath components or the delay of each multipath component provided that the length of the cyclic prefix is larger than that of the channel.

Acknowledgments

The authors would like to express their thanks to International Communications Foundation for its financial support of this work.

References

Appendix: MMSE Solution

Problem: Given a multiuser STBC-OFDM system which is described by the system equation (20)

\[ y = Hx + z \] (A-1)

find two vectors \( w_1^{(v)} \) and \( w_2^{(v)} \) for user \( v \) such that

\[ w_1^{(v)} = \arg \min_{w_1} E \left\{ \left| X_1^{(v)} - w_1^{(v)H} y_1 \right|^2 \right\} \] (A-2)

\[ w_2^{(v)} = \arg \min_{w_2} E \left\{ \left| X_2^{(v)} - w_2^{(v)H} y_2 \right|^2 \right\} \] (A-3)

Solution for \( w_1^{(v)} \)

Let us define the argument of (A-2) as \( A_1 \) and expand it as

\[ A_1 = E \left\{ \left[ X_1^{(v)} - w_1^{(v)H} y_1 \right] \left[ X_1^{(v)} - w_1^{(v)H} y_1 \right]^H \right\} \]

\[ = E \left\{ X_1^{(v)} X_1^{(v)H} \right\} - w_1^{(v)H} E \left\{ y_1 X_1^{(v)} \right\} \]

\[ - E \left\{ X_1^{(v)} y_1^H \right\} w_1^{(v)} + w_1^{(v)H} E \left\{ y_1 y_1^H \right\} w_1^{(v)} \]

Since signals from different users are assumed mutually uncorrelated with one another and with the noise, we have

\[ E \left\{ y_1 X_1^{(v)} \right\} = H^{(v)} E \left\{ x_1^{(v)} X_1^{(v)} \right\} = H^{(v)} [ \xi_0 ^2 \ 0 ]^T \] (A-4)

\[ E \left\{ X_1^{(v)} y_1^H \right\} = [ \xi_0 ^2 \ 0 ] H^{(v)H} \] (A-5)

\[ E \left\{ y_1 y_1^H \right\} = E \left\{ H x_1^{(v)H} H^H \right\} + E \left\{ z z^H \right\} \]

\[ = H A H^{(v)H} + \sigma_1^2 I_{2M} \] (A-6)

where \( A \in \mathbb{C}^{Q \times Q} \)

\[ A \triangleq \begin{bmatrix} A_1 & 0 & \cdots & 0 \\ 0 & A_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & A_Q \end{bmatrix} \] (A-7)

is the diagonal power matrix with elements \( A_q = \xi_q ^2 I_2 \).

Note that

\[ H^{(v)} [ \xi_0 ^2 \ 0 ]^T = \xi_v ^2 \begin{bmatrix} H_{11}^{(v)} & H_{21}^{(v)} & \cdots & H_{M1}^{(v)} \\ H_{12}^{(v)} & H_{22}^{(v)} & \cdots & H_{M2}^{(v)} \end{bmatrix} \]

\[ = \xi_v ^2 \begin{bmatrix} h_1^{(v)H} & h_2^{(v)H} & \cdots & h_{M1}^{(v)H} \\ h_1^{(v)H} & h_2^{(v)H} & \cdots & h_{M2}^{(v)H} \end{bmatrix} \]

\[ \pm \tilde{h}_1^{(v)} \] (A-8)

\[ [ \xi_v ^2 \ 0 ] H^{(v)H} = \xi_v ^2 \begin{bmatrix} H_{11}^{(v)H} & H_{21}^{(v)H} & \cdots & H_{M1}^{(v)H} \\ H_{12}^{(v)H} & H_{22}^{(v)H} & \cdots & H_{M2}^{(v)H} \end{bmatrix} \]

\[ = \xi_v ^2 \begin{bmatrix} h_1^{(v)H} & h_2^{(v)H} & \cdots & h_{M1}^{(v)H} \\ h_1^{(v)H} & h_2^{(v)H} & \cdots & h_{M2}^{(v)H} \end{bmatrix} \]

\[ \pm \tilde{h}_1^{(v)} \] (A-9)

then

\[ A_1 = \xi_v ^2 - \xi_v ^2 h_1^{(v)H} \tilde{h}_1^{(v)} - \tilde{h}_1^{(v)H} \xi_v ^2 h_1^{(v)} \]

\[ + \xi_v ^2 \left( H A H^{(v)H} + \sigma_1^2 I_{2M} \right) w_1^{(v)} \] (A-10)

Taking the derivative of \( A_1 \) in (A-10) with respect to \( w_1^{(v)} \)

while noting that \( \frac{\partial a b}{\partial a} = a^2 \) and \( \frac{\partial b^H a}{\partial b} = 0 \), we have

\[ \frac{\partial A_1}{\partial w_1^{(v)}} = -\tilde{h}_1^{(v)H} + \left( H A H^{(v)H} + \sigma_1^2 I_{2M} \right)^T w_1^{(v)} \] (A-11)

Equating (A-11) to zero gives us

\[ 0 = -\tilde{h}_1^{(v)H} + \left( H A H^{(v)H} + \sigma_1^2 I_{2M} \right)^T w_1^{(v)} \] (A-12)

thus we obtain

\[ w_1^{(v)} = \left( H A H^{(v)H} + \sigma_1^2 I_{2M} \right)^{-1} \tilde{h}_1^{(v)} \] (A-13)

Solution for \( w_2^{(v)} \)

Similar for \( w_2^{(v)} \), we have the following cost function for \( w_2^{(v)} \)

\[ A_2 = E \left\{ \left[ X_2^{(v)} - w_2^{(v)H} y_2 \right] \left[ X_2^{(v)} - w_2^{(v)H} y_2 \right]^H \right\} \]

\[ = E \left\{ \left[ X_2^{(v)} - w_2^{(v)H} y_2 \right] \left[ X_2^{(v)} - w_2^{(v)H} y_2 \right]^H \right\} \]

\[ = E \left\{ X_2^{(v)} X_2^{(v)H} \right\} - w_2^{(v)H} E \left\{ y_2 X_2^{(v)} \right\} \]

\[ - E \left\{ X_2^{(v)} y_2^H \right\} w_2^{(v)} + w_2^{(v)H} E \left\{ y_2 y_2^H \right\} w_2^{(v)} \] (A-14)

Since signals from different users are mutually uncorrelated with each other and with the noise, we have

\[ E \left\{ y_2 X_2^{(v)} \right\} = H^{(v)} E \left\{ x_2^{(v)} X_2^{(v)} \right\} = H^{(v)} [ \xi_0 ^2 \ 0 ]^T \]

\[ = H^{(v)} [ 0 \ 0 ]^T \]

\[ = [ 0 \ 0 ] H^{(v)H} \]

\[ = [ 0 \ 0 ] \]

\[ = \pm \tilde{h}_2^{(v)} \] (A-15)

\[ E \left\{ X_2^{(v)} y_2^H \right\} = E \left\{ X_2^{(v)} y_2^H \right\} H^{(v)H} \]

\[ = [ 0 \ 0 ] \]

\[ = \pm \tilde{h}_2^{(v)} \] (A-16)

As a result, we come up with

\[ A_2 = \xi_v ^2 - \xi_v ^2 h_2^{(v)H} \tilde{h}_2^{(v)} - \tilde{h}_2^{(v)H} \xi_v ^2 h_2^{(v)} \]

\[ + \xi_v ^2 \left( H A H^{(v)H} + \sigma_1^2 I_{2M} \right) w_2^{(v)} \] (A-17)
Taking the derivative of $A_2$ in (A-17) with respect to $w_2^{(ν)}$ and equating it to zero we finally arrive at the solution for $w_2^{(ν)}$

$$w_2^{(ν)} = \left( H \Lambda H^H + \sigma_z^2 I_{2M} \right)^{-1} \hat{h}_2^{(ν)}.$$

(A-18)

Xuan Nam Tran was born on the 8th September, 1971 in Thanh Hoa, Vietnam. He received his bachelor of engineering (BE) degree in radio-electronics from Hanoi University of Technology, Vietnam in 1993, master of engineering (ME) in telecommunications engineering from University of Technology Sydney, Australia in 1998, and doctor of engineering in electronic engineering from The University of Electro-Communications, Japan in 2003. Since November 2003 he has been a research associate at the Information and Communication Systems Group, Department of Information and Communication Engineering, The University of Electro-Communications, Tokyo, Japan. From April 2002 to March 2003 he also worked as a research assistant at Department of Electronic Engineering of the same university. His research interests are in the areas of adaptive antennas, space-time processing, space-time coding and MIMO systems. Dr. Tran is a recipient of the 2003 IEEE AP-S Japan Chapter Young Engineer Award. He is a member of IEEE, and Society of Information Theory and its Applications (SITA).

Yoshio Karasawa received B.E. degree from Yamanashi University in 1973 and M.S. and Dr. Eng. Degrees from Kyoto University in 1977 and 1982, respectively. He joined KDD R&D Labs. in 1977. From July 1993 to July 1997, he was a Department Head of ATR Optical and Radio Communications Res. Labs. and ATR Adaptive Communications Res. Labs., both in Kyoto. From 1997 to 1999, he was a Senior Project Manager of KDD R&D Labs. Currently, he is a professor of the University of Electro-Communications, Tokyo. Since 1977, he has been engaged in studies on wave propagation and radio communication antennas, particularly on theoretical analysis and measurements for wave-propagation phenomena, such as multipath fading in mobile radio systems, tropospheric and ionospheric scintillation, and rain attenuation. His recent interests are in frontier regions bridging “wave propagation” and “digital transmission characteristics” in wideband mobile radio systems and digital signal processing antennas. Dr. Karasawa received the Young Engineers Award from the IEICE of Japan in 1983 and the Meritorious Award on Radio from the Association of Radio Industries and Businesses (ARIB, Japan) in 1998. He is a member of the IEEE and URSI.

Tadashi Fujino was born in Osaka, Japan on 15 July, 1945. He received B.E. and M.E. degrees in electrical engineering and Ph.D. degree in communication engineering from Osaka University, Osaka, Japan, in 1968, 1970, and 1985, respectively. Since April 2000, he has been Professor in wireless communications at the Department of Information and Communication Engineering, The University of Electro-Communications, Tokyo, Japan. Before then, he had been with Mitsubishi Electric Corporation, Tokyo, Japan, since 1970, where he devoted in R&D in the wireless communications area such as digital satellite communications and digital land mobile communications. His major works include the development of 120 Mbit/s QPSK modem and the trellis coded 8-PSK modem to operate at 120 Mbit/s for INTELSAT use, and portable phones PDC and PHS used in Japan and Asia and GSM used in Europe. He wrote a single authored book “Digital mobile communication,” and three coauthored books. He holds over 20 patents. He received Meritorious Award from the ARIB (the Associate of Radio Industries and Businesses of Japan) of MPT of Japan, in 1997. Dr. Fujino is a Fellow of the IEEE. He is also a member of Society of Information Theory and its Applications.